COMPUTER MODEL OF TWO-DIMENSIONAL CONDUCTION AND FORCED CONVECTION IN STORED GRAIN

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A mathematical model of two-dimensional conduction and forced convection in stored grain was developed and verified. Temperatures, moisture contents and deterioration of wheat stored in cylindrical granaries and aerated intermittently can be simulated by operating the model on a computer. Satisfactory agreement was obtained between measured and predicted temperatures and moisture contents in a 40-tonne bulk of utility wheat during periods of no ventilation and ventilation at airflow rates near those commonly recommended for aeration.

INTRODUCTION

There has been a recent trend to the storage of grain on farms in large steel bins (Muir 1980). The temperature of the grain at the centers of large bins may remain for long periods above the minimum required for rapid deterioration by insects, mites, and fungi (Yaciuk et al. 1975). For example, the predicted temperature at the center of a 6-m-diameter bin of wheat harvested on 1 Sept. at 35°C will stay above 20°C until March of the following year (Yaciuk et al. 1975). This slow rate of cooling in large bins along with the early harvest during hot August days (Prasad et al. 1978) may have been the main factors contributing to the outbreak of pests in stored grain in the southern Canadian Prairies in the fall of 1981 (Redekop 1981). If optimum conditions (about 30–35°C) were maintained at the center of a bin of wheat stored in August, two insects (viz. rusty grain beetles or red flour beetles) could multiply to about 500 000 by November (Sinha 1971).

One method of reducing the deterioration of stored grain is to cool it (Sinha 1971) with near-ambient temperature air using an aeration system. An aeration system can also prevent deterioration by reducing the temperature gradients throughout the grain bulk which may cause moisture migration and pockets of mouldy grain.

The condition of aerated grain is highly dependent upon weather conditions, initial grain conditions, airflow rate and other factors. Because there is neither the time nor money to investigate all parameter combinations experimentally, many investigators have resorted to developing mathematical models of the heat and moisture transfer in grain bins. Using historical weather data and numerous combinations of variables, trends in system performance can be quickly derived by conducting experiments with these models on a digital computer.

The objective of this study was to develop a mathematical model that could be used to predict temperatures, moisture contents and deterioration of stored grain. The model was designed to simulate conductive heat transfer in the radial and vertical directions and forced convective heat and moisture transfer in the vertical direction in cylindrical granaries. (Yaciuk et al. 1975). This slow rate of cooling in large bins along with the early harvest during hot August days (Prasad et al. 1978) may have been the main factors contributing to the outbreak of pests in stored grain in the southern Canadian Prairies in the fall of 1981 (Redekop 1981).

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The accuracy of predictions made using mathematical models depends on the adequacy of the relationships used to describe the physical and biological parameters in the grain. Predictions made using a mathematical model are practically useless unless the model has been validated by comparing predicted output with experimentally determined data (Brooker et al. 1974). The model was validated by comparing predicted temperature, moisture content and grain deterioration values with experimental data obtained from an aerated grain bin containing wheat. Although exact predictions are impossible, a model can sufficiently represent the real processes so that useful information can be derived.

This paper describes the development and verification of a model which simulates intermittent aeration of stored wheat on the Canadian Prairies. By altering the equations which are specific to wheat, this model could be modified to predict conditions in a bin containing any hygroscopic granular material.

MATHEMATICAL MODEL

Modelling Intermittent Aeration

Grain aeration systems are usually operated in response to seasonal weather variations, resulting in intermittent fan operation. There are often extended periods when ventilation is not required; therefore, an accurate mathematical model of aeration must describe grain conditions with and without ventilation. Models presently available are designed to simulate either forced convection or conduction (Morey et al. 1976; Pierce and Thompson 1980; Converse et al. 1969; Muir et al. 1980). A model capable of simulating both simultaneously could be developed using analytical methods, deriving relationships for heat and moisture transfer which would apply both with and without airflow. Alternatively, existing modelling methods which describe grain conditions with and without ventilation could be combined into one model. It was this combined model approach which was employed to develop the mathematical model used in this project.

A model to simulate grain conditions in both the vertical and radial directions of cylindrical grain bins was developed for the following reasons. (1) Forced convection through grain results in vertical temperature gradients. These gradients will likely be large because of the low airflow rates required for aeration. (2) Muir et al. (1980) suggested that compared with a one-dimensional model, increased accuracy could be obtained with a two-dimensional conduction model for bins having diameter-to-height ratios of 1.2:1 and greater.

Conduction Model

The conduction component of the com-
The combined model was based on the two-dimensional model developed by Muir et al. (1980) for an unventilated bin. This model considers heat transfer in both the vertical and radial directions of a cylindrical grain bin. Temperatures throughout the bin were assumed to be symmetrical about the vertical axis; heat generation within the grain was assumed to be negligible; and free convection was excluded from the final model.

The cylindrical bin was divided into a finite number of spatial elements in the vertical and radial directions (Fig. 1). Equations capable of predicting the temperatures of each element were developed using a finite-difference method to solve the differential-equation of transient heat transfer (Fourier equation).

The rate of conductive heat flow is

\[ q = -kA \frac{\Delta T}{\Delta x} \]  

(1)

where:

- \( q \) = rate of heat flow (W);  
- \( k \) = thermal conductivity (W/(m·K));  
- \( A \) = cross-sectional area normal to the direction of heat flow (m²); and  
- \( \frac{\Delta T}{\Delta x} \) = temperature gradient in the direction of heat flow, (K/m).

The rate of change in thermal energy contained in a spatial element is

\[ \dot{u} = Vcp \frac{\Delta T}{\Delta t} \]  

(2)

where:

- \( V \) = volume of element (m³);  
- \( c \) = specific heat (J/(kg·K));  
- \( p \) = density (kg/m³); and  
- \( \frac{\Delta T}{\Delta t} \) = change in element temperature during time interval \( \Delta t \) (K/sec).

The volume of five different sizes of elements was considered in developing the equations. Elements at the exterior wall or the floor may consist of two or more materials. For these elements mean values for specific heat and density can be used. Derivations of these equations are presented in detail by Yaciuk (1973).

For any interior spatial element, \( m, n \), the rate of conductive heat flow into the element is

\[ q_{m,n} = k \left[ \left( n\Delta r + \frac{\Delta r}{2} \right) \Delta \theta \Delta z \right] \left( \frac{T_{m+1,n} - T_{m,n}}{\Delta r} \right) + k \left[ \left( n\Delta r - \frac{\Delta r}{2} \right) \Delta \theta \Delta z \right] \left( \frac{T_{m,n+1} - T_{m,n}}{\Delta r} \right) \]  

\[ + k \left( n\Delta r \Delta \theta \Delta z \right) \left( \frac{T_{m,n-1} - T_{m,n}}{\Delta z} \right) \]  

\[ + k \left( n\Delta r \Delta \theta \Delta z \right) \left( \frac{T_{m-1,n} - T_{m,n}}{\Delta z} \right) \]  

(3)

where \( n \) = identification number of spatial element in radial direction (Fig. 1); \( m \) = identification number of spatial element in vertical direction (Fig. 1); \( \Delta r \) = radial distance (m); \( \Delta z \) = vertical distance (m); \( \Delta \theta \) = included angle of bin sector (rad); \( T_{m,n} \) = temperature of element \( m,n \) at time \( r \) (K).

The rate of change in thermal energy stored in any interior spatial element, \( m, n, \) is

\[ \dot{u} = n\Delta \theta \Delta \theta (\Delta r)^2 c_{m,n} \rho_{m,n} (T_{m,n+1} - T_{m,n})/\Delta t \]  

(4)

where \( c_{m,n} \) = mean specific heat of element \( m,n \) (J/(kg·K)); \( \rho_{m,n} \) = mean density of element \( m,n \) (kg/m³); and \( T_{m,n} \) = temperature of element \( m,n \) at time \( r + \Delta t \) (K).

Where thermal properties in the bin are constant, as with the interior elements, dimensionless moduli are defined as

\[ U = \frac{c_p (\Delta r)^2}{k \Delta t} \]  

(5)

and

\[ E = \frac{(\Delta r)^2}{(\Delta z)^2} \]  

(6)

The predicted temperature at the end of time interval \( \Delta t \) is calculated by combining Eqs. 3, 4, 5 and 6 to obtain:

\[ T_{m,n} = \left[ \frac{2n - 1}{2nU} \right] T_{m,n+1} + \left[ \frac{2n - 1}{2nU} \right] T_{m,n-1} + \left[ \frac{E}{U} \right] T_{m-1,n} + \left[ 1 - \frac{2(E + 1)}{U} \right] T_{m,n} \]  

(7)

Equation 7

Specific equations for elements at the periphery of the bin sector are derived similarly (Muir et al. 1980).

For the central-column elements it is assumed that there is no heat transfer across the central axis. At the bottom surface, heat is transferred to an aeration plenum, not to a concrete and soil foundation as was the case in Muir et al. (1980). In developing equations for the top, wall and bottom surface elements, Eq. 3 is modified to include the convective heat transfer from the exterior surface of the element to the surrounding air (Muir et al. 1980; Yaciuk et al. 1975). In addition, the equations for the external wall elements include a term for the radiation heat exchange between the wall and its surroundings.

The radiant heat transfer to the bin wall surface is calculated according to the method presented by Muir et al. (1980) except for the solar radiation components. Coefficients for the average total-radiation striking all sides of a cylindrical bin at
Winnipeg were modified to calculate hourly values:

\[ H_{on} = 0.1152 H_{sv} + 664.9 \frac{H_{on}}{H_b} - 1131 \]  

where \( H_{sv} \) = hourly radiation on a vertical surface \( (J/(m^2-h)) \); \( H_b \) = total radiation on a horizontal surface \( (J/(m^2-h)) \); and \( H_{on} \) = extraterrestrial radiation for the given location \( (J/(m^2-h)) \).

Total solar radiation on a horizontal surface, \( H_s \), was estimated using a model developed for the Canadian Prairies by Won (1977). This model uses readily available hourly meteorological-variables to estimate global radiation. This permits use of this program for locations where hourly global radiation data are not available. The hourly meteorological-variables required are cloud opacity, barometric pressure, dew point and dry bulb temperatures. Extraterrestrial radiation, \( H_{on} \), is calculated using the relationship presented by Won (1977).

**Forced Convection Model**

The equilibrium drying model developed by Thompson (1972) provided the basis for the forced convection component of the combined model. It was used because of its ease of comprehension, efficient use of computer facilities, reported validity, and availability. The model is limited to use when the stored grain and the air passing through it are at near-equilibrium conditions of moisture and temperature. The basic assumptions of this model are (1) Equilibrium is obtained between the air and the grain for the simulation time interval and space increment. (2) Heat and mass transfer between the air and grain are adiabatic; i.e., there is no heat or moisture transfer to or from the surroundings of the grain storage. (3) No hysteresis exists between the absorption and desorption isotherms relating grain equilibrium moisture content to equilibrium relative humidity of the air. (4) No heat or moisture is generated in the grain bulk. Heat and moisture generation might be expected from respiration of the grain, and insect and fungal activity, but these are probably negligible until the rate of deterioration increases to an unacceptable level.

To permit compatibility with the two-dimensional conduction model, this forced convection model was modified to simulate conditions in each of the vertical columns of elements. The number of columns is dependent on the number of conduction nodes used (Fig. 1). During forced convection, conditions in each column were assumed to be independent of those in adjacent columns. To reduce computer time, however, if moisture contents and temperatures of each layer were within specified tolerances (0.5% and 2°C, respectively) of each other, they were averaged, and the grain bin was treated as one column.

**Wheat Deterioration Model**

An accurate mathematical representation of the biological processes contributing to grain quality deterioration has yet to be derived. Quality deterioration in storage is a function of many variables in addition to the most commonly considered ones of grain temperature and moisture content. Some of the other factors which are difficult and perhaps impossible to define using mathematical relationships are: mechanical damage to the grain, grain deterioration prior to harvest and the initial level of infestation by fungi, insects and mites.

Because of the many factors which influence the rate of deterioration, models of this type should be considered approximate. Although their use may not predict absolute safe storage times, relative comparison of the effects of various storage methods on the predicted deterioration can yield useful data from which management recommendations for aeration systems can be based.

The model developed by Fraser and Muir (1980) to predict the allowable safe storage time for wheat was used to assess grain deterioration with and without ventilation. For each time interval the temperature and moisture content of each spatial element are used to calculate the allowable storage time. The proportion of allowable storage time elapsed during the time interval is calculated by dividing the length of time interval by the calculated allowable storage time. This value is added to the proportion of allowable storage time which has already elapsed to obtain an estimate of the total deterioration since harvest. The proportion of allowable storage-time elapsed is expressed as a decimal fraction. A value of 1.0 indicates that the allowable storage time has expired. The model is based on the criterion that spoilage has occurred when seed germination decreases about 5%.

**Additional Data**

Equations for the specific heat of wheat were taken from Muir and Viravanichai (1972). The temperature rise for airflow across an axial-flow fan is a function of the total efficiency of the fan, the static pressure, and the airflow rate. This was calculated using the theoretical equation verified by Metzger et al. (1981). Airflow resistance data for wheat were obtained from ASAE Data D272 (American Society of Agricultural Engineers 1980a). The psychrometric properties — absolute humidity, saturation vapor pressure, and relative humidity — were determined using relationships presented by Wilhelm (1976).

**Simulation Procedure**

The skeleton flow chart (Fig. 2) shows a simplified version of the simulation procedure. After reading the input parameters, simulation begins for each harvest year using historical weather data on tape. Normally simulation begins on a fall harvest date and continues for a maximum of 1 yr. Based on input parameters controlling fan operation, the grain conditions are determined using the conduction or forced convection subroutines and the appropriate time interval. Grain deterioration during each interval is estimated by calculating the additional proportion of allowable storage time elapsed which is then added to that already elapsed.

The cost and operating time required to carry out a simulation on the computer varied greatly depending upon the amount of fan operation. Using the Amdahl 470/V7 computer at the University of Manitoba to perform simulations, fan operation, as modelled by the forced convection component, increased computer demands by a factor of nearly 10 when compared with simulations involving no ventilation. The complete model, written in Fortran notation, and its validation and application, are presented by Metzger (1980).
Figure 2. Skeleton flow chart of the combined computer simulation model.

Figure 3. Cross-section of experimental aeration bin showing the thermocouple and grain-sampling locations.
h on 3 Oct. 1979. The airflow rate was 9.0 (L/sec)/m³. Since this is a much higher airflow rate than required for aeration, the fan was turned off at 1150 h on 5 Oct. As grain temperatures and moisture contents were such that spoilage was not likely, the fan remained off until ambient weather conditions permitted further cooling of the grain.

A baffle to restrict airflow was fabricated and installed on the fan to reduce the airflow to 1.9 (L/sec)/m³. The average temperature rise measured across the baffled fan during the ventilation period was 4.9 ± 1.4°C.

The baffled fan was turned on at 1120 h on 8 Nov. 1979 when average daily air temperatures were less than −10°C. The fan remained on until 0950 h on 21 Nov. 1979. The grain was stored without further ventilation until early February 1980 when it was removed for use as livestock feed.

Using the 1979 weather data on tape and the system parameters for the experimental Glenlea aeration bin, two sets of predictions were made of grain temperatures, moisture contents, and deterioration. One set of predictions began on the bin fill date of 2 Oct. 1979 and included the 2-day period of 9.0 (L/sec)/m³ ventilation. The second set began at 1200 h on 5 Oct. 1979 and did not include the period of high airflow rate. Tape weather data were available for 1979 only, limiting comparisons of measured and predicted data to the period 2 Oct. to 31 Dec. 1979.

Comparison of Measured and Predicted Results

Grain temperatures

Predicted grain temperatures were compared with temperatures measured with thermocouple 11 located 0.43 m from the bin wall, and thermocouple 12, located 0.43 m from the bin center-line. Both locations were 2.2 m above the bin floor (Fig. 3). Predicted temperatures closely followed measured temperatures as shown by the thermocouple 11 data (Fig. 4). The maximum differences were 2.5°C for thermocouple 11 on 3 Nov. and 3.5°C for thermocouple 12 on 31 Dec.

Grain moisture contents

Predicted and measured average grain moisture contents at the three sampling depths were compared, as well as the average of moisture contents throughout the bin. Measured and predicted moisture contents from locations 1 and 2 were averaged to obtain floor level values (Fig. 5), locations 3 and 4 for center values (Fig. 6), and locations 5 and 6 for top values.

Figure 4. Measured and predicted grain temperatures at thermocouple location 11 (Fig. 3), 0.43 m from the bin wall and 2.2 m above the fully perforated floor in a 4.3-m-diameter bin of aerated wheat.

Figure 5. Measured and predicted moisture contents at sample locations 1 and 2 (Fig. 3), 0.2 m above the fully perforated floor in a 4.3-m-diameter bin of aerated wheat.

Figure 6. Measured and predicted moisture contents at sample locations 3 and 4 (Fig. 3), 1.4 m above the fully perforated floor in a 4.3-m-diameter bin of aerated wheat.
values (Fig. 7). All six were averaged to obtain an average moisture content for the grain (Fig. 8).

Floor level moisture content predictions, which included the 9.0 (L/sec)/m³ ventilation period, overestimated moisture losses (Fig. 5). During the initial 2-day period, measured moisture contents dropped 0.5 percentage points, while the model predicted reductions of about 2.2 percentage points. If the high ventilation period is ignored, the measured and predicted values follow more closely. During the 13-day period of 1.9 (L/sec)/m³ ventilation, measured moisture content reductions of 0.6 percentage points occurred. The model predicted reductions of 0.9 percentage points for this period. This is close to the experimental error of about ±0.2% for moisture measurements.

Predictions of average moisture content of the grain bulk followed the measured values closely, even with inclusion of the high-airflow-rate period. Maximum deviations of 0.3 percentage points occurred (Fig. 8).

**Grain deterioration**

Germination tests and fungal counts indicated that grain quality was not reduced significantly during the fall storage period. Germination averaged 98% ranging from 96 to 99%. Fungal counts identified a predominance of those field fungi normally associated with freshly harvested grains.

The predicted average proportion of allowable storage time elapsed was 0.2 after storage for 3 mo. Due largely to the decrease in grain temperature, both ventilation periods decreased the rate of increase of the proportion of allowable storage time elapsed.

**DISCUSSION AND CONCLUSIONS**

**Grain Temperatures**

Grain temperature predictions for the two thermocouple locations examined followed measured values closely (Fig. 4). Accuracy of the strip chart recorder is ±0.5°C. Additional deviations could be due to inaccuracy in the measurement of temperature rise across the fan, air temperature changes within the plenum, incorrect data for the thermal properties of wheat, and variations in weather conditions between the bin location at Glenlea and the Winnipeg weather station.

**Grain Moisture Contents**

Predicted grain moisture contents for the top, center, and floor levels, and for the bin average, compared closely with measured values, if the high 9.0 (L/sec)/m³ ventilation period was not included in the simulation (Figs. 5–8). The initial 2-day period of 9.0 (L/sec)/m³ ventilation resulted in predictions of significantly greater moisture loss at floor level (1.6 percentage points), and slightly greater moisture gains at the center and top levels (0.1–0.5 percentage points) than measured.

These results indicate that equilibrium is not a good assumption at airflow rates as high as 9.0 (L/sec)/m³ using these simulation parameters; however, at 1.9 (L/sec)/m³ moisture content predictions were within experimental error for the 3-mo period used in this comparison. Simulation parameters, which may affect the validity of the equilibrium assumption, include the chosen depth of the convection layers, the simulation time interval, and the air velocity through the grain. For the validation simulations, each of the 10 convection layers was 0.367 m deep, the simulation time interval was 1 h, and the apparent air velocity was 8.24 mm/sec at 9.0 (L/sec)/m³ airflow rate, and 1.74 mm/sec at 1.9 (L/sec)/m³ airflow rate. Improved accuracy at the high airflow rates may be possible by increasing the simulation time interval, or by increasing the convection layer depth by decreasing the number of layers. The air velocity is a function of the
bin dimensions. It may be however, that equilibrium is not a good assumption at the higher airflow rates, and other modifications to the model will be required to improve accuracy.

**Grain Deterioration**

Given the late harvest date and the resulting relatively low grain temperatures and moisture contents, difficulties in maintenance of quality during the fall and winter storage periods were not anticipated. The results of both the fungal and germination quality assessments, and the computer prediction of the proportion of allowable storage time elapsed support this conclusion. Unfortunately, since grain quality deterioration did not reach a critical level, the deterioration model cannot be verified with certainty. Validation of the deterioration model under conditions when deterioration is more likely is required.

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