Natural convection and temperature of stored produce — a theoretical analysis

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Smith, E. A. and Sokhansanj, S. 1990. Natural convection and temperature of stored produce — a theoretical analysis. Can. Agric. Eng. 32: 91-97. The temperature in bins of porous material is affected by the ambient weather conditions. The effect of convection and natural convection in this process is examined. A condition for convection to occur in porous material is derived by an approximate analysis of the system and this condition is confirmed by detailed numerical solutions of the full equations. The analysis shows that for small cereal grains such as wheat, heat transfer is dominated by conduction, but for larger particles the effect of convection is more important. Respiration, and the slow moisture transfer that occur in storage have little effect on the average temperature of the material. However, if convection dominates the energy transport process, the moisture transfer has a noticeable effect on temperature in the regions where the transfer is occurring. But if convection is significant the temperature throughout the bin is not influenced by respiration or slow rate of drying or rewetting. The effect of convection is to mix the temperature more thoroughly so that it is closer to the ambient value.

INTRODUCTION

The quality of agricultural material is influenced by its temperature and moisture so it is important to be able to predict temperature and moisture inside stores. It is a common perception, borne out by limited experimental data, that as a result of heat-induced natural convection a differential moisture distribution occurs within the grain in store. Aggregated high-moisture zones are ideal sites for mould and insect growth, accelerated respiration, and dry matter loss.

The temperature of the stored material is determined by the ambient weather conditions and by the method by which changes in the weather are transferred into the store. A weather factor which has a major effect on the storage temperature is the solar radiation (Jiang and Jofriet 1987). Thus the color of buildings and the material from which they are constructed have an effect (Muir 1973). Annual variations in temperature are also important but short-term daily changes are not (Converse et al. 1973).

Most models which simulate the temperature of stored products assume that thermal conduction is the main form of heat transport (Yaciuk et al. 1975; Jiang and Jofriet 1987). It is known that natural convection occurs in grain silos (Schmidt 1955) but it seems that this does not greatly influence the temperature if only conduction is required to model the heat transfer process. Conduction models have been used to estimate the air flow through beds of grain and the rate of mass transfer (Lo et al. 1975; Muir et al. 1980). These methods are successful in simulating the temperature and moisture content even in systems with complex geometries (Gough 1985). This paper considers whether models based on conduction will always be adequate enough to calculate bed temperatures and if not, to establish when convection is important.

The process of natural convection in a porous medium is described by Combarnous and Bories (1975). Details of both theoretical and experimental work are described. Most of the work does not deal with agricultural conditions. An exception is the study by Beukema et al. (1983) of natural convection when there is a heat source in the porous material. Their model was used to study heat transfer in a rectangular bin and a full three-dimensional analysis was used. The work showed that natural convection influences the cooling of agricultural produce for storage. They found that convection increased the rate of heat transfer and resulted in a steady state temperature which was 11% below the value when no convection was included in the model.

Work by Close and Peck (1986) shows that the techniques used to study heat transfer in natural convection, where there is no mass transfer, can be used to study the situation where there is mass transfer. In their experiment they studied air flow through a packed bed of glass spheres where the spheres were coated with water. The coating was maintained by a flow of water through the bed. The resulting process involved both heat and mass transfer. But Close and Peck (1986) were able to show that the equations which described the process could be put in the same form as the equations which describe heat transfer alone. Thus the same techniques can be used to solve the problem. It is possible that this approach could be used to model natural convection involving heat and mass transfer in grain stores.

The full equations, which describe natural convection in grain beds were derived by Nguyen (1987) and several simulations were presented, including the case in which there is an open space above the grain bed. The equations include the convection transport of moisture but not the transfer by diffusion. The velocity of air during natural convection is very small so moisture transport by diffusion is almost as fast as convective transport (Close and Peck 1986). However, Davidson (1986) showed that reasonable, accurate results can be achieved by ignoring diffusion.

The methods used to solve the conduction equation have been the finite difference method (Yaciuk et al. 1975) and the finite element method (Jiang and Jofriet 1987). For the equations which model both convection and conduction, the main technique used has been the finite difference method. There are several ways of using the finite difference method for this problem. A detailed description of one useful method is given by Datta and Teixeira (1987), for natural convection in canned liquid food, which could be modified for natural convection in a porous medium. The technique involves an upwind differencing method for the convection term. Then using values of temperature and stream function at one time the temperature...
at the end of the next time step is calculated using an implicit
alternating method. Finally the stream function is computed
using a method of successive overrelaxation. Several other,
similar approaches are described by Bejan (1984).

The finite difference method, rather than the finite element
method is probably used because of its greater stability when
a process involves both convection and conduction. But in
natural convection the air velocity is usually very small so insta-
sibility in the numerical method is less likely. The results
presented here show that the finite element method works satis-
factorily and has the advantage that it can more easily deal with
curved boundaries.

In this paper the grain temperature is modelled using equa-
tions describing heat transfer by convection and conduction but
not mass transfer. The work shows why pure conduction models
give good estimates of grain temperature. The conditions under
which the conduction model is not satisfactory are derived. In
particular it is shown that convection strongly affects the tem-
perature when the packed bed is formed of particles with
diameters larger than those of small cereal grains such as wheat.

MATHEMATICAL MODEL

The velocity of the air through the porous medium satisfies two
equations (Beukema et al. 1983)

\[ \nabla \cdot \mathbf{v} = 0 \]  
(1)

(the symbols used are defined in the List of Symbols)

\[ \mathbf{v} = \frac{-k}{\mu} \left\{ \mathbf{v} p - \rho_a \left[ 1 - \beta(T - T_a) \right] \right\} \]  
(2)

where the Boussinesq approximation has been used. This means
that in deriving Eq. 1 it was assumed that the density of the
air \( \rho_a \) was constant while in Eq. 2 the density is a function of
temperature

\[ \rho \approx \rho_a \left[ 1 - \beta(T - T_a) \right] \]

where \( \beta \) is the thermal expansion coefficient of air, \( T_a \) is the
average temperature of the ambient air and \( \rho_a \) is its density.
The values of \( T_a, \rho_a \) and \( \beta \) are assumed to be constant. For this
to be valid, \( T - T_a \) should not be too large \( (\ell < 20^\circ C)\).

Equation 2 is a variation of Darcy’s law. This equation is not
accurate for the air velocity used when drying agricultural
material. But with natural convection the velocity is very small
and Darcy’s law is accurate if the velocity is less than the limiting
velocity, which from Bird et al. (1960) is

\[ \frac{d\rho_a v}{\mu(1-\varepsilon)} < 10 \]

so \( v < 3 \times 10^{-2} \text{ m/s} \)

assuming \( d = 3 \times 10^{-3} \text{ m}, \mu/\rho_a = 1.5 \times 10^{-5} \text{ m}^2/\text{s} \) and
\( \varepsilon = 0.4 \) for air and grain. This is also supported by data on
the pressure drop through wheat at low air velocities (American
Society of Agricultural Engineers (ASAE) Yearbook 1986) which
show that air velocity increases linearly with pressure drop.

Equations 1 and 2 could be used to calculate the pressure \( p \)
and the components of \( \mathbf{v} \), but it is simpler to rewrite Eq. 1 in terms
of a stream function \( \chi \) in cylindrical coordinates (Bird et al. 1960)

\[ \nu_r = \frac{1}{r} \frac{\partial \chi}{\partial z} \quad \text{and} \quad \nu_z = -\frac{1}{r} \frac{\partial \chi}{\partial r} \]  
(3)

These equations automatically satisfy Eq. 1. Then replacing \( \mathbf{v} \)
by \( \chi \) in Eq. 2 and using \( \nabla \times (\nabla \rho) = 0 \) gives

\[ 0 = \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial \chi}{\partial r} \right) + \frac{\partial}{\partial z} \left( \frac{1}{r} \frac{\partial \chi}{\partial z} \right) + \frac{R_m \alpha}{HA} \frac{\partial T}{\partial r} \]  
(4)

where \( Ra = \kappa \rho_a g R_H (\mu \alpha) \) is the Rayleigh number. The value of
this number determines whether or not convection is important.

The energy equation (Beukema et al. 1983) is

\[ \frac{\partial T}{\partial r} + \gamma (\mathbf{v} \cdot \nabla)T = \alpha v^2 T + Q / (C_p)_m \]  
(5)

where \( \gamma = C_p / (C_p)_m \).

The heat capacity of the air in the porous medium is
\( C_p = 1230 \text{ J} / (\text{m}^3 \text{K}) \) (Beukema et al. 1983). The heat capacity
of the mixture of air and porous material is \((C_p)_m\).

The ratio \( \alpha = k/(C_p)_m \) is the thermal diffusivity of the mixture,
where the term \( k \) is the thermal conductivity of the mixture of
air and solid matter. Values of these constants for many agricul-
tural products are recorded in Mohsenin (1980) and the ASAE
Yearbook (1986).

The air and solid matter are assumed to have the same tem-
perature \( T \). Energy flow is controlled by convection, conduc-
tion and respiration. It is assumed that energy involved in mass
transfer can be ignored. This is later shown to be reasonable
for the slow rates of mass transfer which can occur in storage
conditions.

The rate of heat generation by respiration is

\[ Q = 1.072 \times 10^4 Y \]

where \( Y \) is the value of \( g \text{CO}_2 / \text{kg dry matter produced by the}
porous medium. In this paper the value of \( Y \) is calculated using
the formulae quoted by Thompson (1972) for corn, based on
the work of Steele et al. (1969). For wheat, the method was
modified as described by Morey et al. (1981). In evaluating \( Y \)
it was assumed that the moisture content of the solid is uniform
and that its temperature is given by the energy Eq. 5.

Eqs. 4 and 5 were solved for \( \chi \) and \( T \); then Eq. 3 was used to evaluate velocities.

For most of this paper the temperature was calculated for
porous material in a cylindrical bin with height \( h \) and radius
\( R \). This problem is two dimensional with two coordinates \((r, z)\)
and is also symmetrical about the central line \( r = 0 \). The
boundary conditions for \( T \) and \( \chi \) are based on those used by
Converse et al. (1973). The temperature of the air and solid
at the top and sides of the bin was given by

\[ T = T_a + A \sin \left( \frac{2\pi}{360} (t_i - 70) \right) \]  
(6)

while on the bottom of the bin and on the central line \( (r = 0) \)
the temperature gradient normal to the surface is zero. Also,
no air flows across any solid surface or the central line. These
conditions are
\[ \frac{kT}{\rho H} \geq Ra \]

The initial conditions are that \( x = 0 \) and that the temperature on the lines \( r = 0 \) and \( r = R \), to one half of the particle.

\[ \frac{kH}{\rho L} \geq Ra \]

However, the temperature on the line \( \frac{kH}{\rho L} = 0 \) is the same, and the temperature on the line \( \frac{kH}{\rho L} = \frac{H}{\rho L} \). Thus, the condition for convection is important when

\[ \frac{kH}{\rho L} \geq Ra \]

The approximate conditions when conduction and convection are important are derived in this section. To do this, only order of magnitude estimates of the terms in Eqs. 1, 2 and 5 will be used.

For conduction to significantly affect the temperature of the solid material, Eq. 5 says

\[ \frac{kH}{\rho L} \geq Ra \]

For wheat, \( Ra \approx 8 \times 10^7 \) s, where \( a = 1.076 \times 10^{-7} \) m/s and \( R = 3 \) m. This means that ambient temperature variations of the order of a year will influence the temperature of the solid but variations with a shorter timescale, such as diurnal variation, will not. This was shown to be true for bins of wheat by Converse et al. (1973). Because of this, the ambient temperature is modelled by Eq. 6 in which daily variations are ignored.

Equations 1 and 2 show that the velocity can be estimated by

\[ \frac{kH}{\rho L} \geq Ra \]

where it is assumed that \( \frac{kH}{\rho L} \approx A_1 \), and \( A_1 \) is the amplitude of the temperature variation in Eq. 6.

Convection will be important in the energy Eq. 5 if it is as large or larger than the conduction term. This gives

\[ \frac{kH}{\rho L} \geq Ra \]

But from Eq. 1, \( \frac{kH}{\rho L} \approx \frac{H}{\rho L} \)

so

\[ \frac{kH}{\rho L} \geq Ra \]

Thus if \( \frac{kH}{\rho L} \geq Ra \) then convection is important when

\[ \frac{kH}{\rho L} \geq Ra \]

The initial conditions are that \( x = 0 \) and that the temperature of the air and grain inside the bin are equal to the ambient temperature (Eq. 6) when \( t = 0 \), \( \frac{kH}{\rho L} \geq Ra \)

Thus the condition for convection is important when

\[ \frac{kH}{\rho L} \geq Ra \]

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representation because it is simpler and commonly found to be accurate (Zienkiewicz 1977).

The error in the method is proportional to the sum of the error in time and the error in space calculations. For time, the error was proportional to the square of the time step and for the space calculations the error was proportional to $N^{-2}$ where $N$ was the number of elements used. The area was divided into triangular elements and a cubic basis function was used.

Thus, by a combination of reducing the time step and increasing the number of triangles, the error in the solution was reduced. This was done until the value of temperature for one solution was less than 0.05°C different from the solution obtained using the next most accurate solution.

The values of the time step and the number of triangles depend on the problem being solved. But if large velocities and steep gradients occurred typical values were $N = 100$ and time step $\Delta t = 1$ h. But fewer elements and longer time steps could be used in problems where conduction dominates the heat flow process.

**VALIDATION**

To check that the model gives accurate results, two sets of experiments were simulated and the results compared with the observed values.

In the first experiments described by Converse et al. (1973) the temperature of the air/grain mixture was measured in a tall silo of wheat. In this experiment the energy flow was dominated by conduction. The second set of experiments concerned natural convection in a bed of sand saturated with water. The bed was heated uniformly and significant convective flows were produced (Hardee and Nilson 1977).

**Conduction**

In the experiment described by Converse et al. (1973) wheat filled a cylindrical bin with $H = 33.528$ m and $R = 2.743$ m. The temperature of the ambient air was described by Eq. 6 with $T_a = 14.2^\circ$C and $A = 15.3^\circ$C. The initial temperature of the grain was $6.1^\circ$C and the moisture content was 13.3% wet basis. The value of thermal diffusivity $\alpha = 1.076 \times 10^{-7}$ m$^2$/s. For wheat and low airflow the value of $\mu/k$ was $4 \times 10^5$ thus $Ra = 5 \times 10^4$. With this value, the approximate analysis suggests that convective flow will not significantly influence the temperature.

The values of temperature produced by the simulation were very similar to those calculated by Converse et al. (1973). For example the temperature at a radius of 1.524 m (5 ft) are shown in Fig. 1 together with the values simulated by Converse et al. (1973).

When $t_f = 180$ days (summer) in Eq. 6 the calculation showed that the airflow is counter clockwise. At the point $r = (7/8) R (= 2.4$ m), $z = H/2 (= 16.764$ m) the velocity $v = 5.4 \times 10^{-5}$ m/s and at the point $r = (1/8) R$, $z = H/2$ the velocity $v = 1.0 \times 10^{-4}$ m/s. Which confirms that the velocities are very small.

**Convection and conduction**

In the experiment by Hardee and Nilson (1977) a rectangular bed of sand was filled with NaCl water electrolyte. The width of the bed was 0.3 m and the height $L$ was variable. The sides and bottom of the bed were insulated so that there are no heat flux through them. At the top of the bed the temperature was held constant (Fig. 2). Heat was generated electrically in the fluid but in the analysis it was treated as if the heat was generated uniformly throughout the medium. For different values of heat input, the value of the temperature at the bottom of the bed was measured.

Fig. 1. Temperature versus time in a cylindrical bin at a radius of 1.524 m, calculated by Converse et al. (1973) (---), together with the values produced by the Finite Element simulation (-----).  

Fig. 2. Experimental bed of sand and water used by Hardee et al. (1977). The sides and bottom are thermally insulated.

It was established experimentally that there is a simple relationship between the Nusselt number $Nu$ and the Rayleigh number $Rah$, for this heated system. These numbers are defined

$$Nu = \frac{Q L^3}{2k (T_B - T_C)}$$

$$Rah = \frac{\rho_o g \delta_x Q L^3}{\mu \alpha 2k}$$

The Rayleigh number $Rah$ for this system is based on the heat $Q$ input to the system. But it plays a similar role to the Rayleigh number $Ra$, used earlier in Eq. 4, which is based on the variations in the ambient temperature.

The Nusselt number is the ratio of $Q$, the rate at which heat is put into the system, to the rate at which heat is conducted out of the top of the bed. When conduction dominates the process $Nu = 1$ but as convection becomes more important the value of $Nu$ increases. Hardee et al. (1977) show that, in this system, convection is important when $Rah > 32$, but conduction dominates the process when $Rah$ is smaller.

They show experimentally and theoretically that
$$Nu = 1 \text{ when } Ra < 32$$

$$Nu = (Ra/32)^{1/2} \text{ when } Ra > 32$$

which is drawn on Fig. 3.

This process was simulated using Eqs. 1, 2 and 5. The airflow Eqs. 1 and 2 were simplified as before using a stream function $\chi$. In the rectangular coordinates $(x,y)$ of this experiment, the stream function is given by

$$v_x = -\frac{\partial \chi}{\partial y} \text{ and } v_y = \frac{\partial \chi}{\partial x}.$$  

The equations were solved using the method described in the section on Computation Procedure. The results are shown in Fig. 3 together with the values obtained by Hardee and Nilson (1977). The values plotted in Fig. 3 are the steady state values which were obtained by running the simulation until the solution was constant from one time step to the next. The simulated values of the Nusselt number $Nu$ in Fig. 3 are approximately 10% lower than the experimental values. This error in $Nu$ is due to errors in both the temperature and the fluid velocity. When the fluid velocity is low ($Nu = 1$) the error in the temperature is approximately 10%. But if the velocity is larger then the error in the temperature is less than 10%. The error may arise because the steady state was not reached in the simulations or that numerical instabilities occurred in these long running simulations.

It was in this work that the cubic, triangular elements were required to maintain accuracy when large values of heat were put into the system. In the simulation of the previous experiment (Converse et al. 1973) accuracy would have been maintained using linear or quadratic elements.

**RESULTS AND DISCUSSION**

The effect of convection on the temperature and velocity of the air flowing through a porous medium was studied for the cylindrical bin described earlier when modelling the Converse et al. (1973) experiments. For these simulations the radius of the bin was taken as $R = 3$ m. The ambient temperature was given by Eq. 6 with $T_a = 14.2^\circ C$ and $A = 15.3^\circ C$ and the other boundary conditions were given by Eq. 7. Initially ($t_d = 0$) the grain temperature was uniform and equal to the ambient air temperature ($-0.2^\circ C$).

**Temperature**

Solving the equations, using the method described earlier, produced the results shown in Fig. 4. Temperature is shown as a function of increasing convection, represented by $Ra$. The ambient temperature is changing with time and the values of average temperature shown in Fig. 4 are the maximum values in the first year. This always occurred in the 28th week, just after the ambient temperature had passed its maximum value. Similar results occur throughout the year.

From Fig. 4 the average temperature of the $H/R = 1$ bin becomes affected by convection when $Ra > 10^4$. Similarly, for the bin with $H/R = 10$, the temperature increases when $Ra > 10^5$. This agrees with the approximate analysis given earlier.

The average temperature for the bin with $H/R = 10$ is lower than the value for the bin with $H/R = 1$. This is because in the bin with $H/R = 1$, conduction is important both horizontally and vertically. But when $H/R = 10$ only the horizontal conduction is significant. Thus the $H/R = 10$ bin will respond more slowly to the changing ambient temperature.

In Fig. 4 the ambient temperature was increasing so the temperature of the bin with $H/R = 10$ lags behind the temperature in the $H/R = 1$ bin.

For the values used in this paper $Ra/H \sim 1.4 \times 10^3$ for wheat so convection has a small influence on the grain temperature but the temperature will be dominated by conduction. For tall bins ($H/R = 10$) horizontal conduction is the main method of heat transfer as shown by Converse et al. (1973). For squarer bins, both vertical and horizontal conduction are important as shown by Muir et al. (1980).

For agricultural products with a larger diameter, the effect of convection will be more important. Individual temperatures in the porous medium can change significantly due to convection. In Fig 4 the temperature at the centre of the bin ($r = 0, z = H/2$) is shown. For low values of $Ra$, conduction dominates the heat transfer from the high ambient temperature to the low value at the center. But as $Ra$ increases above $10^4$ the central...
temperature approaches the average value because the rapid transport of energy by convection produces a more uniform temperature distribution.

Respiration and moisture transfer

The effect of respiration on the temperature was very small. For the bin with \( H/R = 1 \) the values of temperature are plotted in Fig. 4. In one simulation, respiration was calculated using a moisture content of 10% wet basis and in the other, the moisture content was 19% wet basis. For the wetter grain the temperature was approximately 0.05°C above the temperature of the drier grain; this small difference does not show up in Fig. 4.

The effect of the slow transfer of moisture on the temperature of the material was also small. This was shown by assuming that the mass transfer rate was 2% wet basis in one year, which is typical of reported values for wheat (Schmidt 1955; Muir 1973). To estimate the effect on temperature of this transfer of moisture the following approximate calculation was used. During summer (weeks 12–38) the material was assumed to absorb moisture at the rate of 2% wet basis per year in the area around the center of the bin \( (r = 0, z = H/2) \). This moisture came from the material which was drying in the area around the top, center of the bin. For the rest of the year the process was reversed with drying at the center and rewetting at the top, centre of the bin. This produced average temperatures which were approximately 0.03°C above the average temperature when there was no drying.

Others (Converse et al. 1973; Gough 1985) did not find it necessary to include moisture transfer in order to obtain accurate simulations of temperature in grain bins. When Yaciuk et al. (1975) included the effect of mass transfer they found it did not improve their conduction-dominated model.

The temperature at the center of the bin \( (r = 0, z = H/2) \) which occurred during the drying simulation is shown in Fig. 4. It is approximately 0.5°C warmer than the value when there is no moisture transfer. But this difference is reduced as increased convection produced a more uniform temperature. The conclusion is that the average temperature is not influenced by mass transfer. But the temperature is affected at the place where the mass transfer occurs if conduction dominates the heat transfer.

Velocity

The effect of convection on the velocity of the air through the porous medium is shown in Fig. 5. The bin had dimensions \( H = R = 3 \) m and the moisture content of the material was 10% wet basis so that respiration was negligible. The streamlines when \( Ra = 10^2 \) and when \( Ra = 10^3 \) have a similar shape but the gradient is much larger when \( Ra = 10^5 \).

Figure 5 shows the situation in week 28 (summer) and the air circulates in a counter clockwise direction. When \( Ra = 10^2 \) the velocity at \( r = R, z = H/2 \) was \( 4.0 \times 10^{-6} \) m/s while at \( r = R/8, z = H/2 \) the velocity was \( 2.6 \times 10^{-7} \) m/s. When \( Ra = 10^3 \) the velocity at \( r = R, z = H/2 \) was \( 4.0 \times 10^{-3} \) m/s and at \( r = R/8, z = H/2 \) the velocity was \( 1.8 \times 10^{-4} \) m/s.

For this \( H/R = 1 \) bin the velocity will exceed the \( 3 \times 10^{-2} \) m/s value where the Darcy Law equation is not valid when \( Ra > 10^6 \). So the present analysis cannot be used for greater values of \( Ra \).

CONCLUSIONS

Approximate analysis of the energy and velocity equations show that the temperature of a porous medium is affected by the convective transport of energy if

\[
Ra \geq \left[ 2 + \left( \frac{H}{B} \right)^2 + \left( \frac{R}{H} \right)^2 \right] \frac{1}{2\gamma}
\]

This result is confirmed by numerical simulation of the full equations for porous material in a cylindrical bin.

For small cereals, such as wheat, \( Ra \) is not usually much above this value so there is little influence from convection. But ambient temperatures, bin geometry or loose packing of the cereals could make \( Ra \) larger so that convection would have more effect. For larger particles such as potatoes, oranges or even shelled corn the effect of convection will be more noticeable.

The numerical simulations show that the average temperature of the porous material is not affected by the slow respiration or low rate of moisture transfer that normally occurs in storage bins of wheat. Also when convection dominates the energy transfer process, temperatures throughout the bin are unaffected by slow moisture transfer because convection rapidly

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mixes up the energy density making it more uniform. But if conduction is dominant, larger temperature gradients occur and the local temperature is affected by moisture transfer.

The numerical method used to solve the equations was the finite element method. This performed satisfactorily even when there were relatively large convective flows but greater precision was required for these cases compared with problems where conduction dominated the heat transfer.

LIST OF SYMBOLS

\[ \begin{align*}
A &= \text{amplitude of ambient temperature variations, equation (6)} \\
C_p &= \text{heat capacity of the air} \\
(C_p)_m &= \text{heat capacity of the mixture of air and porous material} \\
d &= \text{diameter of particles which form the porous material} \\
g &= \text{gravitational acceleration} \\
H &= \text{height of the cylindrical bin} \\
k &= \text{thermal conductivity of the mixture of air and porous material} \\
L &= \text{height of the bed of sand in the Hardee et al. (1977) experiment} \\
N &= \text{number of elements used in the finite element method} \\
N_u &= \text{Nusselt number} \\
p &= \text{air pressure} \\
Q &= \text{rate of heat input to a system. Either artificial heat or heat from respiration} \\
R &= \text{radius of the cylindrical bin} \\
Ra &= \text{Rayleigh number} = k \rho_a g \beta \Delta H/(\mu \alpha) \\
Rah &= \text{Rayleigh number for the heated system} = k \rho_a g \beta Q L^3/(\mu \alpha 2k) \\
r &= \text{radial coordinate in the cylindrical coordinate system} \\
T &= \text{temperature of the mixture of air and porous material} \\
T_a &= \text{average temperature of the ambient air} \\
T_B &= \text{temperature at the bottom of the bed of sand} \\
T_C &= \text{temperature of the cold surface at the top of the bed of sand} \\
t &= \text{time in seconds} \\
t_d &= \text{time in days from the beginning of the year} \\
v &= \text{superficial velocity of the air (m}\,\text{/m}^2\text{s}) \\
x &= \text{horizontal coordinate in the rectangular coordinate system} \\
y &= \text{vertical coordinate in the rectangular coordinate system} \\
z &= \text{vertical coordinate in the cylindrical coordinate system} \\
\alpha &= \text{thermal diffusivity of the mixture of air and porous material} \\
\beta &= \text{thermal expansion coefficient of air} \\
\gamma &= \text{ratio of the heat capacities} \\
\delta t &= \text{time scale over which temperature changes significantly} \\
\Delta T &= \text{approximate value of the temperature difference between the ambient temperature and the temperature at the center of the storage bin} \\
\epsilon &= \text{porosity} \\
\kappa &= \text{permeability} \\
\mu &= \text{viscosity} \\
\rho &= \text{density of air} \\
\rho_o &= \text{density of air at temperature } T_o \\
x &= \text{stream function}
\end{align*} \]

References


