A method for measurement of horizontal to vertical pressure ratios of wheat and barley in a circular bin

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INTRODUCTION

The ratio of horizontal to vertical stress at any point in a granular mass is defined as the pressure ratio $K$. This is one of the parameters required for the calculation of stresses that a material exerts on the walls and floor of its container. The three states of stress associated with $K$ are referred to as the active, passive, and at rest states. The first two are the result of wall movement, respectively away and toward the stored material, while the third occurs when an unyielding retaining structure prevents lateral deformation at the periphery of the material, such as in the case of grain bins. The 'at rest' value, which is the subject of this study, lies between the active (lower bound) and passive (upper bound) values, and must be measured for the actual materials to be stored (Blight 1988).

A variety of experimental techniques have been employed for the measurement of $K$ but the results vary considerably from researcher to researcher and indeed from test to test. Many have used the triaxial apparatus (Gudehus 1986; Lohnes et al. 1985; Smith 1981) to measure material parameters that allow the calculation of the pressure ratio, whereas others (Atewologun et al. 1989; Loewer et al. 1977; Clower et al. 1973; Perry and Jangda 1970; Deutsch and Schmidt 1969) have developed techniques for direct measurement of the $K$ ratio. Deutsch and Schmidt (1969) designed pressure cells to measure overpressures on silo walls during discharge. The diaphragms of the cells were instrumented with four miniature strain gauges. During the tests, a drift was experienced as the output was so small at low pressures that it approached the error range of the cells. Perry and Jangda (1970) used a pressure sensitive radio pill, which moved with the free flowing material in model bunkers while measuring the pressure in one direction. However, the pill had a limited range of 0.45 m, and it could not be used inside metal bunkers as the signal transmission was by radio waves. Clower et al. (1973) measured the pressure ratio of granular materials as a function of the vertical confining pressure. Two blades, one oriented horizontally and the other vertically, were placed within the grain mass in a square bin. The blades were withdrawn from the mass while the confining pressure was applied. The ratio of the forces required to overcome the frictional resistance was related to $K$, which was found to be independent of the vertical pressure. Atewologun et al. (1989) developed a diaphragm pressure transducer to measure the normal stresses in a granular medium. This transducer
system was held in place by a rod while the external load was applied to the free surface, thus measurements were made in the direction of the sensors. However, problems were encountered with tilting of the sensors due to bending of the rod, as well as with bridging of the material over the sensors. It is apparent that the pressure ratio is not easily determined because of the problems associated with manouverability, orientation, and size of the measuring instrument. Therefore, this project was undertaken with the specific objectives: (a) to develop a method for accurate experimental determination of the pressure ratio within granular media in deep bins; (b) to test the reliability of the method with respect to various agricultural granular materials; and (c) to compare the experimental results for $K$ with the recommendations in the silo design standards in Canada and the United States.

Figure 1. Orientation of cube sensors.

**MATERIALS AND METHODS**

The pressure measuring apparatus was fabricated by gluing together 6 mm cast acrylic plates to form a cube of 50 mm side. Three pressure sensors (IC Sensors Model 81 - 015G) 18 mm in diameter were fitted flush with the outside surface on three adjacent faces of the cube. The pressure sensors utilized silicon oil coupled with a piezoresistive sensor to monitor the pressure exerted on the stainless steel diaphragm. This oil cushion allowed for the increased protection needed for in-mass measurement. These sensors provided simultaneous measurements of three normal stresses within the grain mass. A schematic diagram of the pressure cube and orientation of the sensors is shown in Fig. 1.

The pressure sensors were calibrated by subjecting each sensor to air pressure (Fig. 2), which was monitored by means of a mercury manometer with an electric voltage output. All sensors were found to provide repeatable results when exposed to air pressure. To ensure consistent response when the load was applied by a granular medium, the sensors were mounted individually, facing vertically upwards at the bottom of a rectangular container, which was then filled with grain and uniformly loaded with weights. The results were repeatable and in agreement with the previous calibration, provided the ratio of sensor diaphragm area (254.5 mm$^2$) to grain surface area was greater than 15. Both materials used in this study, wheat and barley, met the above criterion.

To determine the tilt of the cube, two silicon accelerometers (IC sensors Model 3021) were mounted on the inside walls of the cube, one opposite each of the sensors for measuring pressures in the horizontal or close to the horizontal direction (sensors 2 and 3). This allowed measurement of the rotation of the cube about the X and Y axes, respectively, angles $\theta$ and $\alpha$ in Fig. 1. There was no need to measure the rotation about the vertical axis, since the pressure components can be considered independent of the circumferential coordinate for the axisymmetric case.

The strain-gauge equipped arm of each accelerometer was vertically oriented for maximum sensitivity and supported from the top to avoid hysteresis in the readings. The accelerometers, with arms pointing vertically downwards, were calibrated by rotating the cube in a bracing structure (Fig. 3) and taking readings for small angular changes. To check whether the tilt about one axis affected the accelerometer reading about the other axis, the output voltages from both accelerometers were recorded with tilt taking place in one direction only. The accelerometers were found to be insensitive to cross axis rotation.

A model bin was constructed from a PVC sewer pipe of length 875 mm with inside and outside diameters of 300 mm and 316 mm respectively. The bottom of the bin consisted of a loose-fitting disc supported on a load cell (a strain-tube type pressure transducer), which was made of an extruded tube with strain gauges (Micro-measurements type CAE - 06-250 UW - 120) bonded to its surface to form a full-bridge circuit. A base plate was attached to the load cell for increased stability. A vertical pressure was applied through a rigid plate to the top of the grain mass in the bin by a testing machine (Instron Universal Model 4204). To apply a constant sur-

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**Fig. 2. Calibration of pressure sensors.**

**Fig. 3. Calibration of accelerometers.**
charge loading, a slow cycling speed was specified and the upper and lower limits were set to the same value.

In conducting a test, the instrumented cube was placed at a height of 547 mm along the centreline of the grain mass and subjected to pre-selected increments of load, or decrements of load during the unloading phase. All data were recorded after the displacement had ceased under an existing load. A strain rate of 1 mm/min was chosen since it best represented the actual situation, i.e., the filling rates normally encountered in practice. For further details regarding the evaluation of stress and strain rates, the reader is directed to Law (1990).

The in-mass static stress state, the load carried by the silo base, the vertical pressure applied on top of the sample, and the corresponding settlement were measured in the experiment. Two replications of all measurements were made for each granular material.

The following equations were obtained by transformation of plane stress (Law 1990) to calculate the vertical stress (\(\sigma_v\)) and two horizontal stresses (\(\sigma_{h1}\) and \(\sigma_{h2}\)) at the point of measurement, thus providing two independent values of the pressure ratio.

\[
\sigma_v = \sigma_1 \cos^2 \theta \cos^2 \alpha + \sigma_2 \sin^2 \alpha + \sigma_3 \sin^2 \theta \\
\sigma_{h1} = \sigma_2 \cos^2 \alpha + \sigma_1 \sin^2 \alpha \\
\sigma_{h2} = \sigma_3 \cos^2 \theta + \sigma_1 \sin^2 \theta
\]

where:
- \(\sigma_1\) = normal stress on the Z face,
- \(\sigma_2, \sigma_3\) = normal stresses on the X and Y faces, and
- \(\theta, \alpha\) = rotations about the X and Y axes, respectively (Fig. 1).

In this study the shearing stresses on the cube’s surface have been ignored because their contribution to normal stresses on these planes is negligible for small tilt angles. Equations 2 and 3 yielded two independent horizontal stresses, which were then divided by the same vertical stress (Eq. 1) to obtain two K values at each stress level, thus allowing a comparison of measured values for each test situation.

**RESULTS AND DISCUSSION**

**Load-settlement characteristics**

A series of tests was carried out with wheat and barley at initial densities of about 780 and 690 kg\(\cdot\)m\(^{-3}\), respectively. Load-settlement curves for wheat are presented in Fig. 4 for three loading-unloading cycles. It can be seen that a large part of the settlement remained after the first loading cycle. The second and third loading cycles caused a small degree of additional settlement, which was partly elastic and partly inelastic as with the initial settlement; however, the elastic portion of the settlement was almost equal to the plastic part. Therefore, wheat behaves like a strain-hardening material. The settlement that takes place under load increases the rigidity of the material. Similar results were obtained for barley, but it experienced about 50% more settlement than wheat for the same vertical load, indicating that the individual grains of barley are softer than those of wheat.

![Fig. 4. Load-settlement curves of wheat for three loading-unloading cycles.](image)

**Indirect friction tests**

In the experiments, a known load was applied to the top surface of the specimen and the load exerted by the specimen on the bottom surface was measured with the load cell. From this the product of the coefficient of friction between grain and bin wall, \(\mu\), and the ratio of lateral to vertical pressure \(K\) was determined as a function of bin diameter, \(D\), by

\[
\mu K = 0.25 \beta D
\]

where \(\beta\) is derived from the Janssen’s equation, modified to allow a surcharge load (Jofriet and Negi 1983):

\[
\beta = \frac{1}{z} \ln \left[ \frac{q_0 - \rho g / \beta}{q - \rho g / \beta} \right]
\]

where:
- \(q_0\) = applied pressure at top of the specimen,
- \(q\) = pressure at bottom of the specimen,
- \(\rho\) = mass density,
- \(g\) = gravitational constant, and
- \(z\) = depth of specimen.

As the parameter \(\beta\) occurs on both sides of Eq. 5, a process of successive approximation must be used to obtain a solution, but convergence is rapid.

The results for \(\mu K\) from the experiments are shown in Fig. 5 for barley. The \(\mu K\) values decreased with each of the subsequent loading cycles. This is probably due to a decrease in friction caused by the deposition of a waxy alcohol substance on the container wall (Bucklin et al. 1989), even though the cylinder was cleaned prior to the beginning of each test. The value of \(\mu K\) levelled off at about 0.16 for both
wheat and barley for the initial loading.

The observed variation of the load on the base of the model silo with the load applied on top of the specimen is shown in Fig. 6 for two loading-unloading cycles. The load carried by the wall due to friction is the difference between the applied load and the observed base load. Since the relationships are approximately linear, the percentage of the applied load carried by the base or wall did not change considerably with an increase in the applied load. For these tests, the average percentage of load carried by the base was almost 40%. This depends primarily on the height to diameter ratio of the bin, close to 3 in the present case, roughness of the wall, and to a lesser extent on the properties of the contained material.

**Measured stresses and pressure ratios**

Figure 7 shows the vertical and horizontal stresses measured with the instrumented cube during the loading and unloading of wheat. Because of the closeness in agreement between the two values of horizontal stress obtained at each load level, their averages are plotted in Fig. 7. The loading curves for both the vertical and horizontal stresses exhibited approximately linear relationships. Thus, during loading the pressure ratio was almost constant. However, during unloading the vertical and horizontal stresses decreased non-linearly, causing the calculated value of $K$ to increase with a decrease in the applied pressure. This phenomenon can be observed in Fig. 8 which is a plot of pressure ratio versus applied stress for wheat. The loading portion of the curve remained more or less constant while the unloading portion increased with decreasing load. It appears that the horizontal stress does not dissipate as quickly as the vertical stress. This is because the walls constrain the material and the horizontal stress decays slowly, while with the surface loading plate retreating in the vertical direction, the material expands vertically and rapidly dissipates the stress.

The effect of additional loading cycles on the pressure ratio for wheat is shown in Fig. 9. The pressure ratio increased with each loading cycle, probably due to the increase in density of the material caused by the repeated loading. However, the pressure ratio decreased during the unloading. This led to the closing of the gap between $K$ values corresponding to loading and unloading cycles.

**Comparison with design codes**

The International Silo Association (ISA Standards Committee 1981) specifies that the pressure ratio is a linear function of the vertical stress $q$:

$$K = K_0 (1 + mq)$$  \hspace{1cm} (6)

where $K_0$ and $m$ are experimentally determined material-de-
The pressure ratio is an empirical constant and does not vary with depth of fill, or equivalently, the overburden pressure.

CONCLUSIONS

The apparatus developed for the direct in-mass measurement of the horizontal and vertical stresses within granular media functioned well and provided reliable data for wheat and barley. The settlement experienced by these materials was mostly recoverable upon unloading, and their behavior was similar to that of strain-hardening materials when subjected to repeated loading. The percentage of the applied load carried by the base or wall was essentially independent of the change in surcharge loading. For the model bin with an aspect ratio of 3, the average percentage of load carried by the wall was about 60%.

The average values of pressure ratio were 0.38 for barley and 0.48 for wheat. These values appear to be directly related to the stiffness of the material, since barley experienced about 50% more settlement than wheat for the same vertical load. Further, the pressure ratio increased with each loading cycle, which in turn also increased the density of the material. There was a reasonable agreement between the experimentally determined values of pressure ratio and that of 0.4 recommended by the Canadian Farm Building Code.

The values of the pressure ratio for wheat and barley remained quite constant under an increasing stress level. Hence, contrary to some beliefs (e.g. ISA Standards Committee 1981), the pressure ratio is an empirical constant and does not vary with the depth of fill, or equivalently, the overburden pressure.

ACKNOWLEDGEMENT

The work reported herein was carried out with the financial assistance of the Natural Sciences and Engineering Research Council through an operating grant.

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