

# Analytical and numerical models for predicting soil forces on narrow tillage tools - A review

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Kushwaha, R.L., Chi, L. and Shen, J. 1993. **Analytical and numerical models for predicting soil forces on narrow tillage tools - A review.** *Can. Agric. Eng.* 35:183-193. During the last three decades, a number of analytical and numerical models for predicting soil forces on narrow tillage tools have been developed. These models are reviewed in this paper. The discussion of analytical models is focused on the soil failure zone and force equation, while the discussion of numerical models is focused on the soil constitutive equation and stiffness matrix. The limitations and problems associated with the models in predicting soil forces on tillage tools are also discussed.

Au cours des trois dernières décennies, quelques modèles analytiques et numériques se sont développés pour prédire les forces du sol sur les outils de labourages étroites. Ce papier revue ces modèles. La discussion des modèles analytiques se concentre sur la zone de sol failli et l'équation de force, cependant la discussion des modèles numériques se concentre sur l'équation du sol constitutif et la matrice de dureté. En plus, la recherche y comprends les problèmes et les limitation associé avec les modèles pour prédire les forces du sol sur les outils de labourage.

## INTRODUCTION

Tillage is a procedure of breaking and loosening of soil. Large amounts of energy are consumed during tillage operations because of high draft forces. Draft mainly depends upon the soil properties, tool geometry, and travel speed. During the last three decades, several three dimensional analytical models have been developed based on the results from experimental work and Terzaghi's passive earth pressure theory (Terzaghi 1943). In these models, a soil failure pattern was proposed and soil force equations were derived from the proposed failure zone. As the computer became more and more accessible, several numerical models were also developed to study the soil-tillage tool interaction. The objective of this paper is to review the existing soil cutting models for narrow tillage tools.

## REVIEW OF MODELS

### Two-dimensional analytical soil cutting models

The critical soil cutting model was first developed for two-dimensional soil cutting caused by a wide blade based on Terzaghi's passive earth pressure theory (Terzaghi 1943). According to Terzaghi's theory, a failure zone is assumed to exist ahead of a cutting blade and the soil in the failure zone

is assumed to be in the critical failure state. The slope-line theory is then applied to predict the soil forces.

A finite difference method was proposed by Sokolovski (1960) and Harr (1966) to calculate the shape of the slope line and failure area. To obtain a simpler solution, a semi-empirical failure zone was earlier suggested by Terzaghi (1943) for two dimensional soil failure, which consisted of a Rankine passive zone (Rankine 1857) and a complex shear zone bounded by part of a logarithmic spiral curve, as shown in Fig. 1. The resulting force on the blade was calculated by assuming static equilibrium along the boundary and determining the position of O (Fig.1) to provide a minimum value (Terzaghi 1943; Hettiaratchi et al. 1966).

Osman (1964) studied the factors affecting the soil forces in soil cutting by using dimensional analysis. Reece (1965) proposed a general earth pressure equation as:

$$P = (\gamma d^2 N_r + cdN_c + qdN_q) w \quad (1)$$

(See NOMENCLATURE for definition of symbols.)

Based on the logarithmic spiral failure zone, Hettiaratchi and Reece (1974) developed a set of charts to determine the dimensionless factors in Eq. 1. These dimensionless  $N$ -factors are functions of rake angle, soil internal friction, and soil-metal friction. Equation 1 has been widely used for predicting the soil force acting on a wide blade.

### Three-dimensional analytical soil cutting models

Soil cutting under narrow tillage tools is usually a three dimensional problem. The soil failure geometry of three di-

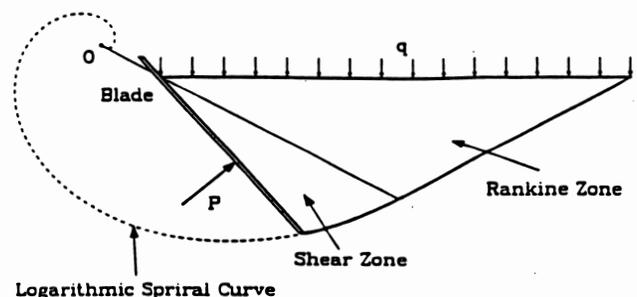


Fig. 1. Logarithmic spiral failure zone.

mensional soil cutting is more complicated than that of a two-dimensional problem. There is no unique solution for the failure pattern and the shape of slip lines for a narrow tillage tool. Several different models have been proposed to evaluate the force required for soil failure.

**Payne's model (1956)** The first three-dimensional soil failure model was developed by Payne (1956) through the study of the soil failure patterns obtained in a series of field and laboratory tests. By observing the top surface soil heave during tillage, a failure zone was proposed as shown in Fig. 2. The failure zone included a triangular centre wedge, a centre crescent, and two side blocks (called wings of the crescent). Further experiments showed that the shape of the failure zone changed with the geometry of tools such as rake angle, depth, and width (Payne and Tanner 1959). Extensive tests were conducted by Payne and Tanner (1959) to study the effects of the failure zone size and the rake angle on the draft force. However, no equations were developed to evaluate the draft force for narrow tillage tools.

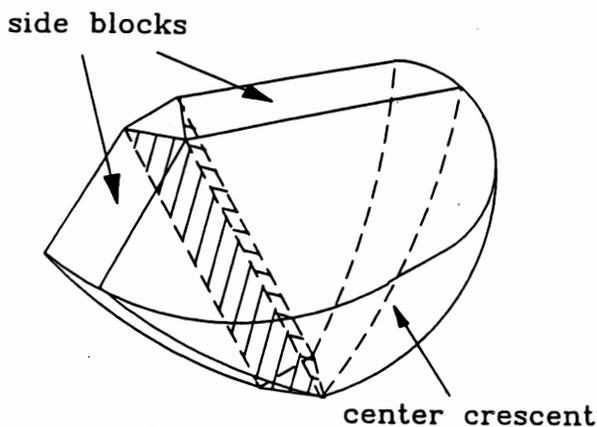


Fig. 2. Failure zone of Payne's model.

**O'Callaghan-Farrelly model (1964)** Following Payne's work, O'Callaghan and Farrelly (1964) carried out a number of field tests for three different soil conditions. At that time, Terzaghi's passive earth pressure theory was widely used in civil engineering to calculate safety factors in structural design. Therefore, O'Callaghan and Farrelly attempted to adapt this classical soil mechanics theory to develop an equation to calculate the draft force during tillage operations. Based on the field observations carried out with a vertical flat blade, a soil failure model was proposed as shown in Fig. 3. The model consisted of a forward failure above the critical depth and a horizontal failure under the critical depth (Fig. 3). The critical depth was equal to the tool width for a smooth blade and half its width for a free surface with a normal restraint. For a shallow blade operating above the critical depth, a draft equation was developed by applying Terzaghi's method (Terzaghi 1943), which is given by:

$$H_s = w (cdN_c + \gamma d^2 N_\gamma) \quad (2a)$$

For a deep blade, the force required for horizontal failure was included as:

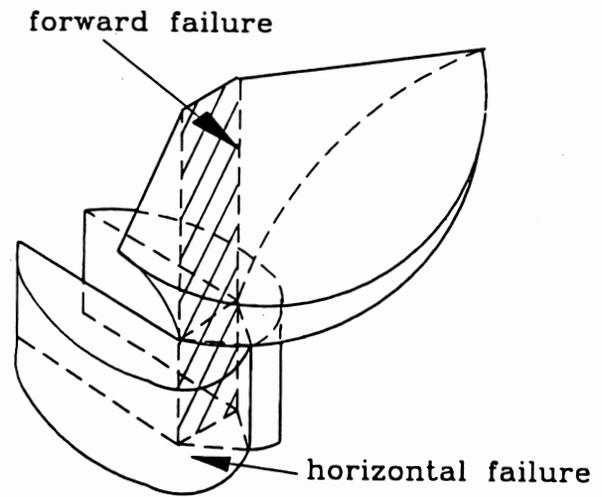


Fig. 3. Failure zone of the O'Callaghan-Farrelly model.

$$H_d = \frac{cw(d-kw)}{\tan\phi} \left[ \tan^2 \left( \frac{\pi}{4} + \frac{\phi}{2} \right) e^{\pi \tan\phi} - 1 \right] + H_s \quad (2b)$$

The comparison between results obtained from the proposed equation and the test data (O'Callaghan and Farrelly 1964) were generally good except for a slight under prediction for the hardest soil tested. The O'Callaghan-Farrelly Model was developed prior to the well known Hettiaratchi-Reece two dimensional model. This model did not include the effect of adhesion and external friction between the soil and the tool surface. During comparison, the gravity term in Eq. 2 was also neglected because of the small masses of soil. In addition, the comparison was only conducted for a vertical blade.

**Hettiaratchi-Reece model (1967)** After developing the well known two dimensional soil failure equation (Eq. 1), Hettiaratchi and Reece (1967) proposed a three dimensional soil failure model. In this model, the soil failure configuration was divided into forward failure, ahead of the soil-tool interface, and transverse failure, the horizontal transverse movement of the soil away from the centre line of the interface, as shown in Fig. 4.

The total force on the tool was determined as a resultant of the forces from forward failure and transverse failure. The force contributed by the forward failure was found simply by using the two dimensional soil failure equation (Eq. 1). The force required for transverse failure was similar to the expression derived by O'Callaghan and Farrelly (1964) except that a gravitational component was included. The total model can be summarized by:

$$P_f = (\gamma d^2 N_\gamma + cdN_c + qdN_q) w \quad (3a)$$

$$P_t = \left[ \gamma \left( d_e + \frac{q}{\gamma} \right)^2 w N_{s\gamma} + cwd_e N_{sc} \right] K_a \quad (3b)$$

$$H = P_f \sin(\alpha + \delta) + P_t \sin \alpha + C_a d \cos \alpha \quad (3c)$$

$$V = P_f \cos(\alpha + \delta) + P_t \cos \alpha + C_a d \quad (3d)$$

The Hettiaratchi-Reece model included the effects of soil

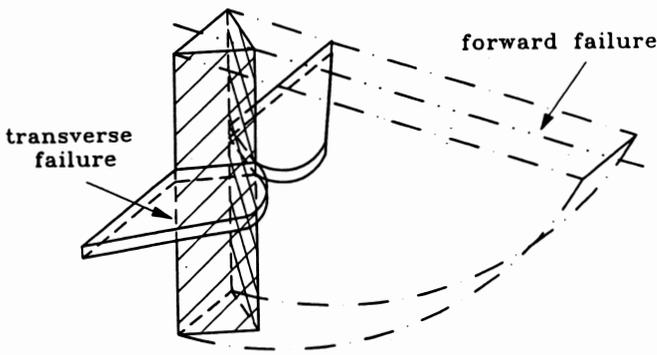


Fig. 4. Failure zone of the Hettiaratchi-Reece model.

properties, soil-metal properties, and tool geometry such as rake angle, depth, and width. However, the model was found to over predict the draft forces (Grisso and Perumpral 1985).

**Godwin-Spoor model (1977)** Godwin and Spoor (1977) studied the soil failure pattern with narrow tillage tines. Two separate failure patterns were proposed: (1) three-dimensional crescent failure above the critical depth, and (2) two-dimensional lateral (horizontal) failure below the critical depth. For the three dimensional crescent soil failure, a failure model was proposed as a parallel centre wedge flanked with two curved side crescents, as shown in Fig. 5. The lateral failure below critical depth was essentially similar to the horizontal failure proposed by O'Callaghan and Farrelly (1964) as well as Hettiaratchi and Reece (1967).

The total force was determined as the resultant of forces acting on all three sections. As with the Hettiaratchi-Reece Model, Eq. 1 and  $N$  factors for soil failure were used to calculate the force for the centre wedge. Besides, the two dimensional force equation was also used for a small element cut from the side crescents (Fig. 5) as:

$$dP_q = (\gamma d_c^2 N_\gamma + cd_c N_c + q d_c N_q) \frac{rd\eta}{2} \quad (4a)$$

An integration method was applied to evaluate the total

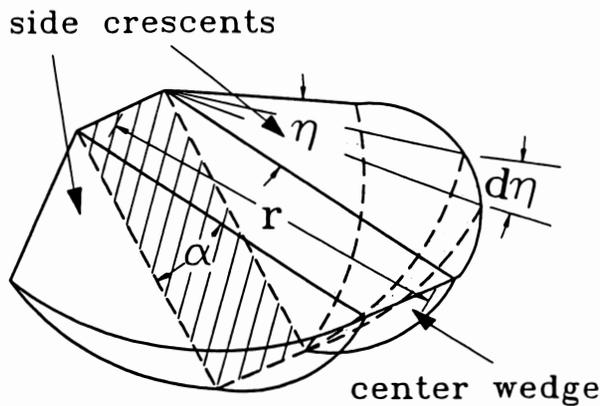


Fig. 5. Failure zone of the Godwin-Spoor model.

force on the side crescents. To simplify the integration, the failure boundary on the top surface was assumed to be circular. The draft and vertical forces for three dimensional crescent failure above the critical depth are given by:

$$H_t = [\gamma d_c^2 N_\gamma + cd_c N_c + qd_c N_q] [w + r \sin \eta] \sin(\alpha + \delta) + C_a wd_c [N_a \sin(\alpha + \delta) + \cos \alpha] \quad (4b)$$

$$V_t = -[\gamma d_c^2 N_\gamma + cd_c N_c + qd_c N_q] [w + r \sin \eta] \cos(\alpha + \delta) - C_a wd_c [N_a \cos(\alpha + \delta) + \sin \alpha] \quad (4c)$$

where:

$\eta$  = extended angle of side crescent (Fig. 5), which is given as:

$$\eta = \cos^{-1} \left( \frac{d_c \cos \alpha}{r} \right) \quad (4d)$$

For lateral soil failure, the force was developed as:

$$H_l = w [cN'_c (d - 2d_c) + 0.5 (1 - \sin \phi) \gamma w N'_\gamma (d^2 - d_c^2)] \quad (4e)$$

The total draft force for a deep blade is given as:

$$H = H_t + H_l \quad (4f)$$

Use of the Godwin-Spoor model required prior knowledge of the rupture distance (Fig. 5). Godwin and Spoor (1977) developed a graph using the information from Payne (1956), Payne and Tanner (1959), and Hettiaratchi and Reece (1967) to describe the relationship between the distance ratio (rupture distance/depth) and tool angle. However, the determination of the rupture distance ( $r$ ) was generally difficult.

**McKyes-Ali model (1977)** In the McKyes-Ali model (McKyes and Ali 1977), a failure wedge was also proposed ahead of the cutting blade, as shown in Fig. 6. Similar to the Godwin-Spoor model, the failure wedge was composed of a

side crescents

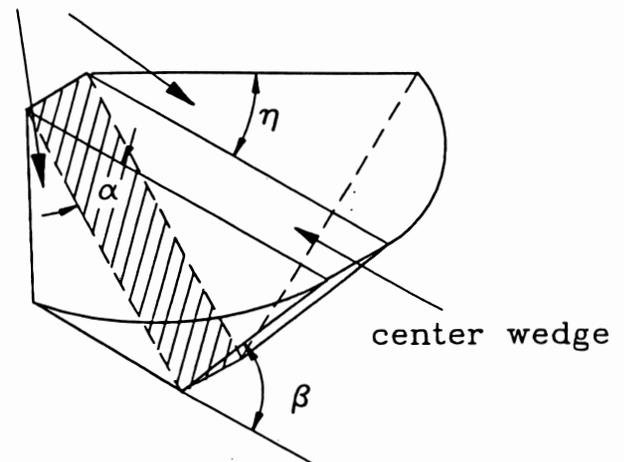


Fig. 6. Single wedge failure zone of the McKyes-Ali model.

centre wedge and two side crescents. The only difference in failure shape was that the bottom surface of the centre wedge was assumed to be a plane and the bottom of the side crescent was assumed to be a straight line (Fig. 6).

The models reviewed in forgoing sections used two dimensional slip line theory to evaluate the forces. McKyes and Ali (1977) sought a new approach to simplify the problem. In the McKyes-Ali model, the forces on each section were determined by applying the mechanics of equilibrium directly rather than using the equation and  $N$  factors of two dimensional soil failure. A flat bottom plane of the centre wedge and the straight line at the bottom of the crescents enabled defining the direction of the reaction forces at the bottom of the failure zone. Forces contributed by the centre wedge and side wedges were also considered. Finally, the proposed draft equation was similar to Eq. 1. However, the  $N$ -factors were re-evaluated for three dimensional soil failure. The factors are given by the following equations:

$$N_{\gamma H} = \frac{\frac{r}{2d} [1 + \frac{2r}{3w} \sin \eta]}{\cot(\alpha + \delta) + \cot(\beta + \phi)} \quad (5a)$$

$$N_{cH} = \frac{[1 + \cot \beta \cos(\beta + \phi)] [1 + \frac{r}{w} \sin \eta]}{\cot(\alpha + \delta) + \cot(\beta + \phi)} \quad (5b)$$

$$N_{qH} = \frac{\frac{r}{d} [1 + \frac{r}{w} \sin \eta]}{\cot(\alpha + \delta) + \cot(\beta + \phi)} \quad (5c)$$

where  $r$  is rupture distance which is given as:

$$r = d (\cot \beta + \cot \alpha) \quad (5d)$$

Each of the dimensionless factors (Eq. 5a to Eq. 5c) was a function of angle  $\beta$  as shown in Fig. 6. Angle  $\beta$  was determined by minimizing the factor  $N_{\gamma H}$  (the factor for the gravity term in the draft). Angle  $\beta$  obtained from this process was used to compute the remaining  $N$ -factors. Compared to the Godwin-Spoor model, the McKyes-Ali model is easier to use and does not require prior knowledge of the rupture distance. McKyes (1985) published a set of charts to determine the  $N$  factors in Eq. 5a through Eq. 5c for some values of internal friction angle  $\phi$  and external friction angle  $\delta$ . However, a simple program can be written based on Eq. 5a to Eq. 5c to evaluate the  $N$  factors.

McKyes and Ali (1977) compared the  $N$  factors in Eq. 5a to Eq. 5c with the  $N$  factors for two dimensional soil cutting by setting  $w = \infty$ . The results showed a very good agreement for a smooth blade ( $\delta=0$ ). For a rough blade with a rake angle greater than  $90 - \phi$ , the  $N$  factors in Eq. 5a to Eq. 5c were much higher than those from two dimensional soil cutting. Therefore, a two wedge model was proposed for a rough blade with large rake angle, as shown in Fig. 7.

**Perumpral-Grisso-Desai model (1983)** Perumpral et al. (1983) proposed another three dimensional soil cutting model for narrow tillage tools. The model replaced the two side crescents flanking the centre wedge by a set of two forces acting on the side of the centre wedge as shown in Fig. 8. The

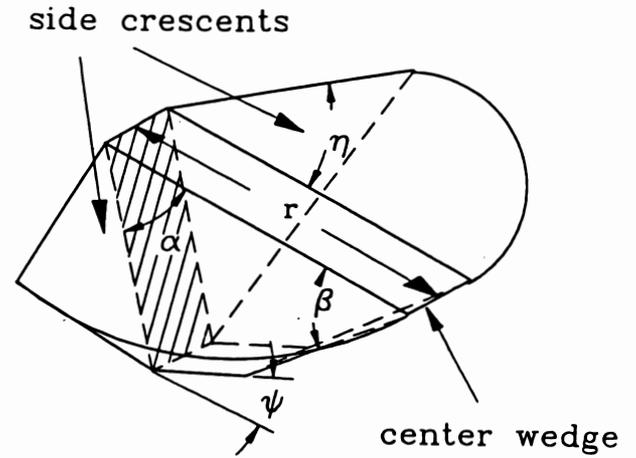


Fig. 7. Double wedges failure zone of the McKyes-Ali model.

authors claimed that the model had the same failure zone as the McKyes-Ali model; however, since the soil weight of the two side crescents was not considered and the side planes of the centre wedge were treated as slip planes, the actual failure zone of Perumpral's model only included a centre wedge. Similar to the model by McKyes and Ali, the bottom slip surface was assumed to be straight. The model also assumed that the soil heaved ahead of the tools (Fig. 8).

The total reaction force was determined by considering the equilibrium of all forces acting on the centre wedge. The force equation was later rewritten into a form similar to Eq. 1:

$$P = w [\gamma d^2 N_{\gamma} + c d N_c + C_a d N_a] \quad (6a)$$

and

$$N_{\gamma} = \frac{\frac{A}{w d^2} [2 K_o z \sin \phi + w \sin(\phi + \beta)]}{\sin(\beta + \alpha + \phi + \delta)} \quad (6b)$$

$$N_c = \frac{\cos \phi [\frac{2A}{w d} + \frac{1}{\sin \beta}]}{\sin(\beta + \alpha + \phi + \delta)} \quad (6c)$$

$$N_a = \frac{-[1 + \frac{h}{d}] \cos(\beta + \alpha + \phi)}{\sin(\beta + \alpha + \phi + \delta)} \quad (6d)$$

where:

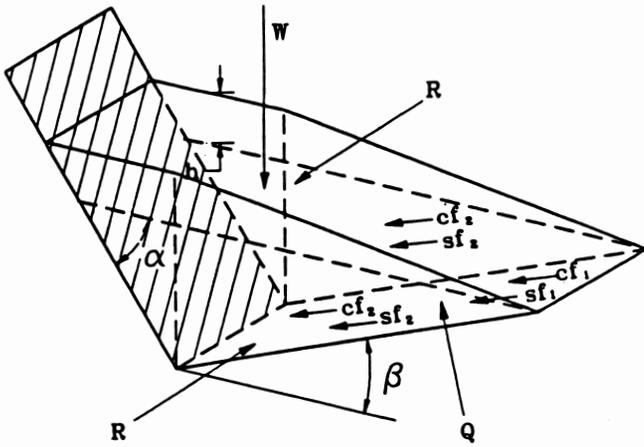
$K_o$  = coefficient of earth pressure at rest, which is given as:

$$K_o = 1 - \sin \phi \quad (6e)$$

$z$  = average depth at which the centroid of the failure wedge is located from soil surface (m).

$$z = \frac{1}{3} (d + h) \quad (6f)$$

Equation 6f was found to be a function of angle  $\beta$  (Fig. 8). Angle  $\beta$  was determined by minimizing the total force  $P$ . A computer program was developed to minimize the force  $P$  and determine the failure plane angle  $\beta$ .



**Fig. 8. Failure zone of the Perumpral-Grisso-Desai model.**

**Swick-Perumpral Model (1988)** All the above models are static models in which the effect of travel speed was not considered. Swick and Perumpral (1988) proposed a dynamic soil cutting model which included the effect of travel speed. The failure zone of the model, similar to the McKyes-Ali model, consisted of a centre wedge and two side crescents with a straight rupture plane at the bottom. In the Godwin-Spoor and McKyes-Ali models, the extreme outer points of the side crescent were assumed to lie in a vertical plane passing through the forward tip of the tool. It was found that this assumption over-predicted the size of the side crescents (Swick and Perumpral 1988). Therefore, based on the observations from soil bin tests, the extended angle  $\eta$  was proposed as a function of the rupture distance and the rake angle (Fig. 9) as:

$$\eta = \frac{\sin^{-1}(-6.03 + 0.46r + 0.0904\alpha)}{r} \quad (7a)$$

The force equation of the model was derived in the same way as that for the McKyes-Ali model except that an acceleration force was added to account for the effect of travel speed. From the equilibrium equation for the centre wedge, the forces on the centre wedge are derived as:

$$P_1 = \frac{\frac{C_a d \cos(\alpha + \phi + \beta)}{\sin \alpha} + \left(\frac{\gamma dr}{2} + qr\right) \sin(\phi + \beta)}{\sin(\alpha + \phi + \beta + \delta)} + \frac{\left(\frac{cd}{\sin \phi} + f_{al}\right) \cos \phi}{\sin(\alpha + \phi + \beta + \delta)} \quad (7b)$$

where:

$$f_{al} = \frac{\gamma d v^2 \sin \alpha}{g \sin(\alpha + \beta)} \quad (7c)$$

The force contribution from the side crescents was derived by considering the equilibrium of a small slice of the side crescents and then integrating this force for the whole side crescents. The tool force of one side crescent is obtained as:

$$P_2 = \frac{\left(\frac{\gamma d r^2}{6} + \frac{q r^2}{2}\right) \sin(\phi + \beta) \sin \eta}{\sin(\alpha + \phi + \beta + \delta)} w + \frac{f_{a2} \cos \phi \left(\frac{\eta}{2} + \frac{\sin 2\eta}{4}\right) + \frac{c d r \cos \phi \sin \eta}{\sin \phi}}{\sin(\alpha + \phi + \beta + \delta)} w \quad (7d)$$

where:

$$f_{a2} = \frac{\gamma d r v^2 \sin \alpha}{2 g \sin(\alpha + \beta)} \quad (7e)$$

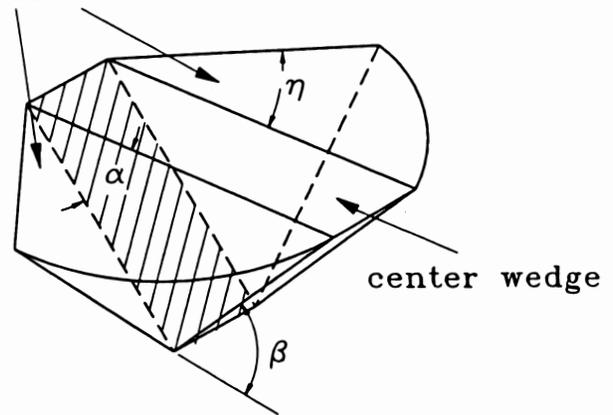
$\eta$  = side crescents extended angle (Fig. 9) given by Eq. 7a.

The total force from the centre wedge and two side crescents is expressed as:

$$P = P_1 + 2 P_2 \quad (7f)$$

The total force is a function of the rupture angle  $\beta$  (Fig. 9). According to the passive earth pressure theory, passive failure takes place when the resistance exerted by the soil wedge is a minimum. The wedge creating minimum resistance was found by minimizing total force  $P$  with respect to the rupture angle  $\beta$ . A numerical procedure was developed to evaluate the total forces.

side crescents



**Fig. 9. Failure zone of the Swick-Perumpral model.**

**Zeng-Yao model (1992)** Zeng and Yao (1992) developed another dynamic soil cutting model which included the acceleration and damping effect on the basis of their studies on the relation between soil shear strength and shear strain rate (Zeng and Yao 1991), and the relation between soil-metal friction and sliding speed (Yao and Zeng 1990). The failure zone of the model was similar to that of the McKyes-Ali model. One major difference between these two models is that the Zeng-Yao model requires prior knowledge of failure shear strain for determining the position of the shear failure boundary.

In this model, the total draft  $P_X$  was divided into five components: compressive force of soil along the board  $P_G$ , side-edge shear force  $P_{SH}$ , inertia force of soil in acceleration

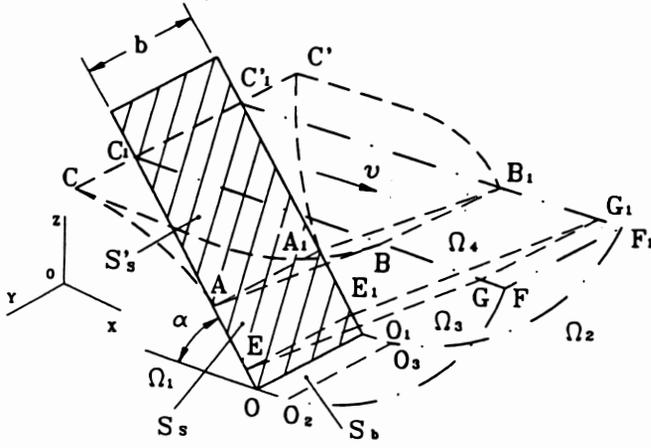


Fig. 10. Failure zone of the Zeng-Yao model.

$P_A$ , bottom-edge cutting force  $P_C$ , and frictional force along the cutting board surface  $P_F$ , as the following:

$$P_x = P_G \sin \alpha + (P_{SH} + P_A) \cos \beta_1 + P_F \cos \alpha + P_C \quad (8a)$$

The total compressive force normal to the board is:

$$P_G = 0.5 b (H_z + L_2) (1 - \sin \phi) f \left[ \frac{1 - \sin \beta}{\sin (\alpha + \beta_1)} \right] \quad (8b)$$

Applying the equation of soil shear strength as a function of shear rate and normal pressure, the side-edged shear force can be expressed as:

$$P_{SH} = e^{C_1} L_2 \dot{\gamma}^2 \frac{[1 + C_4 P_o (1 - \sin \phi)]^{C_3 + 1} - 1}{C_4 P_o (1 + G) (1 - \sin \phi)} \quad (8c)$$

Assuming a linear variation in speed and no direction change, the total inertia force can be expressed as:

$$P_A = b H_z \frac{D_w V^2 \sin \alpha}{g \sin^2 (\alpha + \beta_1)} \quad (8d)$$

The bottom edge cutting force and friction force along the cutting board surface are expressed as:

$$P_C = b Q_o t^{n-1} H_z^m$$

$$P_F = c' + A' \ln [V \sin \beta_1 / \sin (\alpha + \beta_1)] b H_z + P_n \tan \phi_\alpha \quad (8e)$$

### Numerical soil cutting models

The continuous increase of computer running speed and memory during recent years has allowed many investigators to use different types of numerical methods, especially the finite element method (FEM), to analyze the cutting process of tillage tools (Chi and Kushwaha 1989, 1990, 1991; Eldin et al. 1990; Liu and Hou 1985; Wang and Gee-Clough 1991; Xie and Zhang 1985; Yong and Hanna 1977). The general governing equation for FEM analysis in three-dimensional

cases is given as:

$$[M] \{\ddot{a}\} + [V] \{\dot{a}\} + [K] \{a\} + \{f\} = 0$$

$$[K] = \sum_{e=1}^{n_e} \iiint [B]^T [C] [B] dx dy dz \quad (9)$$

Several types of constitutive models were adopted or developed for soil and soil-metal interface to be used in finite element analysis.

### Constitutive models for soil

**Duncan-Chang model** For agricultural soils, a hyperbolic stress-strain model developed by Duncan and Chang (1970) has been adopted by most researchers in their FEM applications. In the model, the tangent modulus is expressed as follows:

$$E_s = K_{so} \left[ \frac{\sigma_3}{P_a} \right]^{n_s} \left[ 1 - \frac{R_{sf} (\sigma_1 - \sigma_3)}{(\sigma_1 - \sigma_3)_f} \right]^2 \quad (10)$$

**Elasto-plastic model** Qiu and Yu (1986) developed an elasto-plastic constitutive model for unsaturated soil. In this model, the shear yield locus is assumed to be a parabola and the yield function  $f_1$  can be expressed as:

$$f_1 = B p_1 + q_1^2 - D = 0 \quad (11a)$$

where:

$$p_1 = (\sigma_1 + 2\sigma_3)/3;$$

$$q_1 = \sigma_1 - \sigma_3$$

The plastic cap was assumed to be part of an ellipse and its yield function was deduced from the theory of revised Cam Clay model.

The plastic potential equation was assumed as:

$$dW_p = p_1 \left[ (d\epsilon_v^p)^2 + (M d\bar{\epsilon}^p)^2 \right]^{0.5} \quad (11b)$$

The yield locus equation can be expressed by an ellipsoidal function:

$$\frac{p_1}{P_o} = \frac{M^2}{(M^2 + n^2)} \quad (11c)$$

where:

$$n = q_1 / p_1.$$

The hardening parameter  $H_1$  proposed by Zienkiewicz and Pande (1977) was:

$$H_1 = (\lambda_1 - k_1) \ln \left( \frac{p_1}{p_2} \right) \quad (11d)$$

The yield function ( $f_2$ ) of the plastic yield stress state was:

$$f_2 = (\lambda - K) \ln \left[ \frac{p(M^2 + n^2)}{p_1 M^2} \right] \quad (11e)$$

**Huang Model (1983)** Liu and Hou (1985) used an elastoplastic model proposed by Huang Wenxi (1983) to conduct FEM analysis of the soil tillage process. In this model, the shear failure surface is expressed as:

$$F_C = q_{oc} - CC - TFP_{oc} \quad (12a)$$

The yield surface was assumed to be a proportional ellipse surface.

$$F_R = \left( \frac{P - H_1}{KH_1} \right)^2 = \left( \frac{q}{KRH_1} \right)^2 - 1 = 0 \quad (12b)$$

The hardening parameter was expressed as:

$$H_1 = \frac{M_5 P_a}{1 + K}$$

$$\left[ \exp \frac{\epsilon_{oc}^p - M_3 \bar{\epsilon}_{oc}^p + \sqrt{(\epsilon_{oc}^p - M_3 \bar{\epsilon}_{oc}^p)^2 - 4 M_2 (\bar{\epsilon}^p)^2}}{2 M_4} - 1 \right] \quad (12c)$$

**Kushwaha-Shen model (1993)** The above three models do not account for the dynamic effect. Kushwaha and Shen (1993) refined the Duncan-Chang model to include the speed effect.

Yu and Shen (1988) established a relation between soil strain rate and stress by means of statistical mechanics as:

$$\ln \dot{\epsilon} = \alpha_2 + \beta_2 \sigma \quad (13a)$$

Combining Eq. 9 and Eq. 12a, the following expression of modified tangent modulus was proposed to account for loading rate effect:

$$E_s = K_{so} P_a \left[ \frac{\sigma_3}{P_a} \right]^{n_s} \left[ 1 - \frac{R_{sf}(\sigma_1 - \sigma_3)}{(\sigma_1 - \sigma_3)_{fo} [1 + B_s \ln(\frac{\dot{\epsilon}}{\dot{\epsilon}_0})]} \right]^2 \quad (13b)$$

### Constitutive model for soil-metal interface

**Clough-Duncan model (1971)** The above four models are used to simulate elements inside the soil body. For the elements at the soil-metal interface, the Clough-Duncan Model was adopted by Chi and Kushwaha (1990). The tangent modulus, according to the model, can be written as:

$$E_i = K_i P_a \left[ \frac{\sigma_n}{P_a} \right]^{n_i} \left[ 1 - \frac{R_{if} \tau}{\tau_f} \right]^2 \quad (14)$$

**Kushwaha-Shen model (1993)** According to the results of high-speed direct shear test, Kushwaha and Shen developed a dynamic constitutive relation for the soil-metal interface. In the model, the tangent modulus was expressed by:

$$E_i = K_i P_a \left[ \frac{\sigma_n}{P_a} \right]^{n_i} \left[ 1 - \frac{R_{if} \tau}{\tau_{fo} [1 + B_i \ln(\frac{\dot{\gamma}_s}{\dot{\gamma}_{so}})]} \right]^2 \quad (15)$$

### Element model for soil-metal interface

Due to specific characteristics of the soil-metal interface, a special element is needed to describe the interaction between soil and metal.

**Joint element model (1968)** Goodman and Taylor (1968) developed a type of interface element called a joint element. In this model, the stiffness matrix is expressed as:

$$[K] = \frac{1}{6} \begin{bmatrix} 2K_s & 0 & 1K_s & 0 & -1K_s & 0 & -2K_s & 0 \\ 0 & 2K_n & 0 & 1K_n & 0 & -1K_n & 0 & -2K_n \\ 1K_s & 0 & 2K_s & 0 & -2s & 0 & -1K_s & 0 \\ 0 & 1K_n & 0 & 2K_n & 0 & 2K_n & 0 & -1K_n \\ -1K_s & 0 & -2K_s & 0 & 2K_s & 0 & 1K_s & 0 \\ 0 & -1K_n & 0 & -2K_n & 0 & 2K_n & 0 & 1K_n \\ -2K_s & 0 & -1K_s & 0 & 1K_s & 0 & 2K_s & 0 \\ 0 & -2K_n & 0 & -1K_n & 0 & 1K_n & 0 & 2K_n \end{bmatrix} \quad (16)$$

**Thin-layer element (1984)** Desai et al. (1984) developed the constitutive model of a thin-layer element. In this model, the element constitutive matrix is expressed as:

$$[c^e]_i = \begin{bmatrix} C_1 & C_2 & C_2 & 0 & 0 & 0 \\ C_2 & C_1 & C_2 & 0 & 0 & 0 \\ C_2 & C_2 & C_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & G_{i1} & 0 & 0 \\ 0 & 0 & 0 & 0 & G_{i2} & 0 \\ 0 & 0 & 0 & 0 & 0 & G_{i3} \end{bmatrix} \quad (17a)$$

where:

$$C_1 = \frac{E_i (1 - \nu)}{(1 + \nu) (1 - 2\nu)} \quad (17b)$$

$$C_2 = \frac{E_i \nu}{(1 + \nu) (1 - 2\nu)}$$

$E_i$  is determined by Eq. 13.

**Friction element model (1985)** Liu and Hou (1985) developed one kind of 2-node friction element. The stiffness matrix of the element can be written as:

$$[K] = \begin{bmatrix} K_s & 0 & 0 & -K_s & 0 & 0 \\ 0 & K_n & 0 & 0 & -K_n & 0 \\ 0 & 0 & K_t & 0 & 0 & -K_t \\ -K_s & 0 & 0 & K_s & 0 & 0 \\ 0 & -K_n & 0 & 0 & K_n & 0 \\ 0 & 0 & -K_t & 0 & 0 & K_t \end{bmatrix} \quad (18)$$

## DISCUSSION

In all analytical models except the Pane's model, soil force equations were developed to calculate draft and vertical force. These equations are generally simple to use. The values of dimensionless factors in the O'Callaghan-Farrelly, Hettiaratchi-Reece and Godwin-Spoor models can be found in the charts developed by Hettiaratchi and Reece (1974). However, the Godwin-Spoor model needs a prior knowledge of rupture distance  $r$ . McKyes (1985) also developed a series of charts to evaluate the N-factors in their model. For the Perumpral-Grisso-Desai model and Swick-Perumpral model, a simple program can be written to determine the rupture angle by minimizing the total tool force. For the Zeng-Yao model, it requires prior knowledge of the failure shear strain for determining the shear failure boundary.

The analytical models generally performed well, based upon the model verification conducted individually by their own authors. Comparisons between some models were also conducted by some researchers. The over-prediction of draft forces of the Hettiaratchi-Reece model was reported in some cases (Grisso and Perumpral 1985). The over-prediction of the Hettiaratchi-Reece model was possibly caused by the assumption of the forward failure and transverse failure at the same time in which the effect of the centre wedge might be considered twice. The Godwin-Spoor, McKyes-Ali, and Perumpral-Grisso-Desai models generally gave an accurate prediction to the forces. However, since the Perumpral-Grisso-Desai model ignored the soil weight of the side crescents, the predicted forces were slightly lower than those predicted by the Godwin-Spoor model and the McKyes-Ali model for a narrow blade (Grisso and Perumpral 1985). The Godwin-Spoor model and the McKyes-Ali model over-predicted the disturbed area (Swick and Perumpral 1988).

In all analytical models, a failure zone was proposed. Some simplification of the failure zone was made in order to develop the force equation, such as circular side crescents (the Godwin-Spoor model, the McKyes-Ali model, the Perumpral-Grisso-Desai model, and the Swick-Perumpral model) and straight bottom rupture plane (the McKyes-Ali model, the Perumpral-Grisso-Desai model, and the Swick-Perumpral model). The prediction accuracy of forces is directly related to the accuracy of the proposed failure zone. In addition, the tool shape, such as sweep and curved blades, may affect the shape and size of the soil failure zone. The analytical models are not capable of considering these effects. Therefore, the models can not provide enough information to guide the optimum design of a tillage tool.

The finite element method showed more flexibility to simulate the tillage operation under different tool shapes. It not only calculated the soil force but also provided a progressive failure zone, soil stress field, soil displacement field, soil velocity field, soil acceleration field, and stress distribution on the tool surface. For example, Fig.11 shows the calculated soil failure pattern from 3-D finite element analysis (Chi and Kushwaha 1990). Information from the FEM models can be used to better understand the soil-tool interaction during tillage. However, there still are some limitations and problems associated with the finite element method. All finite element models require a constitutive relationship of material. The constitutive relationship of agricultural soil is not

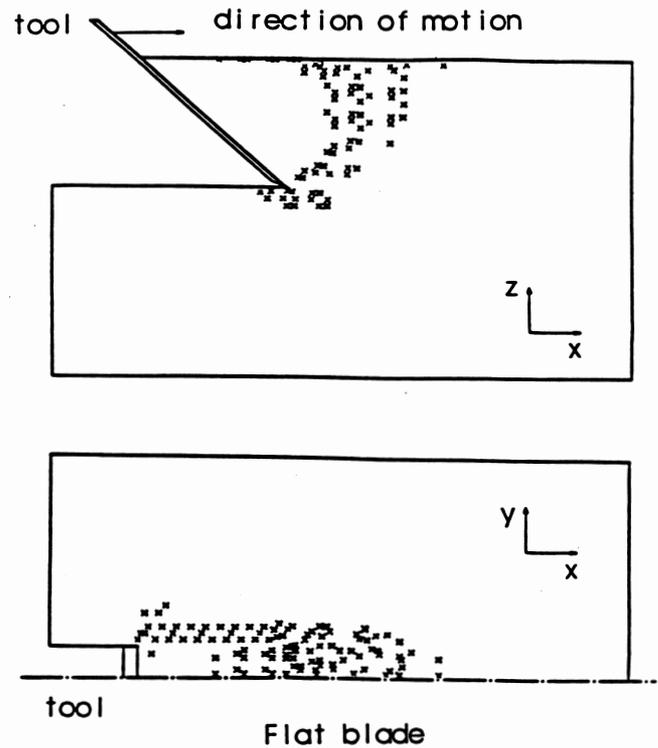


Fig. 11. Soil failure with a flat blade from 3-D finite element analysis.

fully understood yet. Finite element analysis requires a working knowledge of the finite element method and computer programming. The nonlinear 3-D finite element analysis of a narrow tillage tool usually requires substantial computing resources including a main frame computer or workstation and long computing time.

Among static soil models, the Duncan-Chang Constitutive Model was adopted by most investigators for its simplicity and consistency with the test results of agricultural soils. The elasto-plastic model developed by Qiu and Yu requires additional plastic yield criterion and work hardening law. Therefore, it is relatively complex to be used in FEM analysis. The yield function and hardening law of the Huang model were determined directly by experimentation and thus avoid any assumptions. The disadvantage of this model is that it needs equipment more complex than a conventional triaxial apparatus to investigate the effect of stress path. The Kushwaha-Shen Model is a refinement of the Duncan-Chang Model to include a damping effect. The relation between soil strain rate and stress was developed by means of statistical mechanics and indoor laboratory tests.

Of all soil-metal interface models, the Clough-Duncan Model was the most popular constitutive model to simulate soil-metal interaction. Based on this model, three different interface-element models were used broadly.

The first interface-element model was the joint-element model originally developed by Goodman and Taylor (1968). The element formulation was derived on the basis of relative nodal displacements of the soil elements surrounding the interface element and the thickness of the element was often assumed to be zero. There are several drawbacks to this kind

of element. Joint element can only describe the relation between shear stress and relative displacement. When shear stress exceeds the limit of traction criterion, either shear stress  $\tau$  can be reduced or normal stress  $\sigma$  can be increased to meet the need of the friction criterion; thus, a different solution can be obtained, i.e. the solution is uncertain. In addition, interpenetration of nodes may occur after elements are subjected to compressive forces.

The second interface-element model was that of Desai et al. (1984) who designed a kind of thin layer element. The form of the constitutive matrix and the expression of tangential modulus for this element are very similar to those of soil elements with the Duncan-Chang Model. As a result, it is easy to develop a computer program of thin-layer elements in reference to the program of soil elements with the Duncan-Chang model.

The third interface-element model is that of Liu and Hou (1985) who developed one kind of 2-node finite element. The main advantage of this element is the simplicity of the element form. If  $K_n$  is specified as a very large value, the interpenetration of nodes can be avoided. Besides, it is easy to regenerate the elements after large displacement occurred. The disadvantage of the friction element is that the internal stress of friction element has to be determined by the summation of internal stresses of surrounding elements. This causes many extra calculations.

### SUMMARY AND CONCLUSIONS

Several analytical models were reviewed in this paper. All the traditional models proposed a failure zone and derived equations based upon the proposed failure zone. The analytical models are simple to use. However, these models can not include the effects of sweep and curved blades. Therefore, application of these models is limited for optimum design of a tillage tool.

The finite element method has the capability of simulating different tool shapes and dynamic effect of travel speed. The finite element method also provided additional information such as: force, soil stress, soil displacement, etc. This information could help to understand the soil-tool interaction during tillage. However, this method is time-consuming in data preprocessing, programming, and calculations.

At the present time, the application of finite element analysis is limited by lack of an accurate soil constitutive model which can account for the nonlinearity, elasto-plastic deformation, anisotropy, and dynamic properties of agricultural soils.

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## NOMENCLATURE

- {*a*} Displacement vector (m)  
 { $\dot{a}$ } Velocity vector (m/s)  
 { $\ddot{a}$ } Acceleration vector (m/s<sup>2</sup>)  
*A* Side area of failure wedge (m<sup>2</sup>)  
*b* Cutting width (m)  
*B* Coefficient of Eq. 11a  
*B<sub>i</sub>* Coefficient of Eq. 15  
*B<sub>S</sub>* Coefficient of Eq. 13b  
 [*B*] Element strain matrix  
*c* Soil cohesion (Pa)  
*c<sub>a</sub>* Soil-metal adhesion (Pa)  
*C', A', φ<sub>a</sub>* Soil-metal dynamic friction parameters (Yao and Zeng 1990)  
*C<sub>1</sub>, C<sub>2</sub>, C<sub>3</sub>, C<sub>4</sub>* Soil dynamic shear strength parameters (Zeng and Yao 1991)  
*CC* Test parameter of Eq. 12a  
 [*C*] Element constitutive matrix  
*d* Tool operating depth (m)  
*d<sub>c</sub>* Critical depth (m)  
*d<sub>e</sub>* Effective depth of transverse failure (m)  
*D* Coefficient of Eq. 11a  
*D<sub>w</sub>* Soil wet bulk density (kg/m<sup>3</sup>)  
*E<sub>i</sub>* Tangent modulus of interface (Pa)  
*E<sub>s</sub>* Tangent modulus of soil (Pa)  
 {*f*} Force vector (*N*)  
*f<sub>al</sub>* Acceleration force on center wedge (*N*)  
*f<sub>a2</sub>* Acceleration force on side crescent (*N*)  
*g* Gravitational constant (9.81 m/s<sup>2</sup>)  
*h* Height of soil heave in front of the tool at failure (m)  
*H* Draft on tillage tool (*N*)  
*H<sub>1</sub>* Hardening parameter  
*H<sub>d</sub>* Draft for deep blade (*N*)  
*H<sub>l</sub>* Draft from lateral soil failure (*N*)  
*H<sub>s</sub>* Draft for shallow blade (*N*)  
*H<sub>t</sub>* Draft above critical depth (*N*)  
*H<sub>Z</sub>* Cutting depth (m)  
*k* Ratio of critical depth to width  
*k<sub>1</sub>* Test constant of Eq. 11.  
*K, R, M<sub>2</sub>, M<sub>3</sub>, M<sub>4</sub>, M<sub>5</sub>* Parameters determined from the experiment for Eq. 12  
*K<sub>a</sub>* Tine "inclination factor"  
*K<sub>i</sub>* Coefficient of Clough-Duncan Eq. 14  
*K<sub>n</sub>* Normal stiffness (N/m)  
*K<sub>o</sub>* Coefficient of earth pressure at rest  
*K<sub>s</sub>, K<sub>t</sub>* Shear stiffness (N/m)  
*K<sub>so</sub>* Coefficient of Duncan-Chang Eq. 10  
 [*K*] Stiffness matrix  
*M* Coefficient of Eq. 11b  
 [*M*] Mass matrix (kg)

$n_e$	Total number of elements in structure	$\alpha$	Rake angle
$n_i$	Coefficient of Clough-Duncan Eq. 14	$\alpha_2$	Coefficient of Eq. 13a
$n_s$	Coefficient of Duncan-Chang Eq. 10	$\beta$	Failure plane angle
$N_\gamma, N_c, N_q, N_a, N_{s\gamma}, N_{sc}, N_{\gamma H}, N_{cH}, N_{qH}, N'_\gamma, N'_c$	Dimensionless factors	$\beta_2$	Coefficient of Eq. 13a
$p_o$	Reference size of yield locus (Pa)	$\beta_1$	Angle of absolute velocity of the soil slice with the ground surface
$P$	Total force on tillage tool (N)	$\gamma$	Unit weight of soil (N/m <sup>3</sup> )
$P_a$	Atmosphere pressure (Pa)	$\dot{\gamma}_s$	Actual shear strain rate (s <sup>-1</sup> )
$P_A$	Total initial force (N)	$\dot{\gamma}_{so}$	Maximum shear strain rate at direct shear box
$P_C$	Bottom edge cutting force (N)	$\delta$	External friction angle
$P_f$	Force from forward failure (N)	$\dot{\epsilon}$	Actual strain rate in soil (s <sup>-1</sup> )
$P_F$	Friction along cutting board surface (N)	$\dot{\epsilon}_o$	Maximum strain rate at triaxial apparatus
$P_G$	Total compressive force normal to the board (N)	$\epsilon_{oc}^p$	Plastic component of octahedral normal strain
$P_t$	Force from transverse failure (N)	$\bar{\epsilon}_{oc}^p$	Plastic component of octahedral shear strain
$P_x$	Total draft (N)	$d\bar{\epsilon}^p$	Plastic shear strain
$P_{oc}$	Octahedral normal stress (Pa)	$d\epsilon_{v,p}$	Plastic volume strain
$P_{SH}$	Side-edged shear force (N)	$\lambda_1$	Test constant of Eq. 11d
$q$	Surcharge pressure on surface (Pa)	$\nu$	Poisson ratio
$q_{oc}$	Octahedral shear stress (Pa)	$\sigma$	Stress or strength (Pa)
$r$	Rupture distance (m)	$\sigma_n$	Normal stress at interface (Pa)
$R_{if}$	Failure ratio at soil-metal interface	$\sigma_1$	Major principal stress in soil (Pa)
$R_{sf}$	Failure ratio of soil	$\sigma_3$	Minor principal stress in soil (Pa)
$TF$	Test parameters of Eq. 12a	$(\sigma_1 - \sigma_3)_f$	Shear strength (Pa)
[V]	Damping matrix	$(\sigma_1 - \sigma_3)_{f0}$	Shear strength obtained from conventional triaxial test (Pa)
$v$	Travel speed of tool (m/s)	$\tau$	Actual shear stress (Pa)
$V$	Vertical force on tillage tool (N)	$\tau_f$	Shear strength at interface (Pa)
$V_t$	Vertical force from three dimensional soil failure above critical depth (N)	$\phi$	Internal soil friction angle
$W$	Tool width (m)		
$W_p$	Plastic potential (Pa)		
$Z$	Average depth where the centroid of failure wedge is located from soil surface (m)		